MESON DECAYS IN AN EFFECTIVE PERTURBATIVE QCD MODEL

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Abstract: In a continuation of the examination of an effective perturbative QCD approach to some low-energy phenomena of nuclear and particle physics, we consider the decays of mesons into two other mesons. We include S-state and P-state (excited) mesons of the standard quark model description of mesons. Simple effective propagators are used for the colored vector (e.g. gluon-like) boson and the results are compared to experiment.

1. Introduction

In recent years great progress has been made in our understanding of strong interactions in the framework of quantum chromodynamics (QCD) ¹. Applications have been made to nuclear forces ², to pion scattering on nuclei ³, and to a variety of other phenomena. In the same context, we used an effective perturbative QCD approach to describe NN annihilation ⁴). Previously, we briefly described the decays of the light vector mesons in the same model ⁵). Here we generalize this approach to include the two-meson decays of heavier mesons, including those involving quarks in an excited (P-) state. In our model the interaction between the quarks responsible for the reaction is assumed to occur due to the exchange of a single colored vector particle representing one or more gluons. The propagator for this confined particle as well as the coupling of the particle (called “gluon” henceforth) to quarks are taken to be effective ones. Since the “gluon” is confined, its propagator may not be that of a free vector particle. In this article we take two limiting examples of an effective propagator. The coupling of a single gluon is known to be a decreasing function of momentum transfer or an increasing one with distance; here we take a constant value for the coupling of our vector particle. At sufficiently high momentum transfers, the coupling constant of gluons to quarks is weak and perturbation theory applies, but at low momentum transfers the coupling constant is large and perturbation theory breaks down. In practical calculations one tries to overcome such difficulties by introducing confinement mechanisms such as bags or potentials. We

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assume that the strong coupling constant is primarily effective in giving constituent quarks their masses and in confining them, but that the reaction under study occurs due to an effective single gluon-like exchange. The model is related to the impulse approximation in ordinary nuclear physics. That is, we describe hadrons as nonrelativistic constituent quarks confined by a harmonic oscillator potential, which provides the bound quark wave functions, and the decays to occur through an effective transition operator which can be obtained from perturbative QCD. The nonrelativistic approach can be questioned for the light quarks, but we believe that, at this stage, a relativistic approach is not called for. We thus make the following approximations, in addition to a nonrelativistic model:

(i) The confining potential is that of a harmonic oscillator.

(ii) The interaction responsible for the decay is an effective one represented by one colored vector particle or "gluon" exchange. We assume that multigluon exchanges are partially included but that the rest are represented by a scalar interaction, responsible primarily for confinement and the masses of the constituent quarks. These effects are neglected in the decay. The strong coupling constant of a single "gluon" depends on the momentum transfer. For sufficiently high momenta it is given by

\[ \alpha_s(q^2) = \frac{4\pi}{(11 - \frac{2}{3} n_f) \ln (q^2/\Lambda^2)}, \]

where \( n_f \) is the number of quark flavors and the scale factor \( \Lambda = 100-400 \text{ MeV} \). We use an effective constant coupling \( \alpha_s \).

(iii) We neglect long-range effects on the decays. We assume that all mesons are \( q\bar{q} \) states. We do not include pionic degrees of freedom to avoid double-counting.

(iv) We assume constant constituent quark masses for all mesons, 330 MeV for up and down quarks and 580 MeV for strange quarks.

(v) For the "gluon" (we omit the quotes henceforth) propagator we consider two extremes. In the first one we take the propagator for a free gluon and neglect its energy relative to three-momentum, \( q^2 = -q^2 \), which gives rise to a coulombic interaction. In the second case we use an effective constant value which is fixed by the energy carried by the propagator and neglect the three-momentum transfer. This gives rise to a \( \delta \)-function interaction.

2. Theory and results

The basic processes responsible for the decay of the mesons are \( q \to q q \bar{q} \) and \( \bar{q} \to q \bar{q} \bar{q} \), as shown in fig. 1. In the following, we limit our discussion to the first of these processes, but we include both of them in our calculations. This interaction has been considered by us previously for \( N\bar{N} \) annihilation. The nonrelativistic reduction of the amplitudes associated with the diagram up to first order in quark
momenta, leads to

$$v(q,p_1) = -g_s \frac{\lambda_1 \cdot \lambda_3}{4} \frac{1}{\omega_q^2 - q^2} \left[ \frac{\sigma_3 \cdot q}{2m_1} + \frac{1}{m_1} \right] \left[ \frac{i\sigma_1 \cdot \sigma_3 \cdot q}{2m_1} - \frac{\sigma_3 \cdot p_1}{m_1} \right],$$

(2)

where $q = p_1 - p'_1$, $\omega_q = E_{p_1} - E_{p'_1}$, $\sigma_i(m_i)$ is the spin (mass) of quark $i$, $g_s$ is the strong coupling constant and $\frac{1}{4} \lambda_1 \cdot \lambda_3$ is the color factor. In the limit of equal quark masses, the local terms ($\propto q$) in eq. (2) give no contribution to the decays.

As already mentioned, we consider two extreme cases for the gluon propagator, i.e. $\omega_q = 0$ (case A), and $\omega_q \approx 2m_3$ with $q \approx 0$ (case B). With these approximations eq. (2) can be written as $v(r,x_i)$ in coordinate space,

$$v^A(r,x_i) = -ia_s \frac{\lambda_1 \cdot \lambda_3}{4} \frac{1}{2r} \left[ \frac{\sigma_3 \cdot \sigma_3}{m_3} + \frac{1}{m_3} \right] \left[ \frac{i\sigma_1 \cdot \sigma_3}{m_3} - \frac{\sigma_3 \cdot \nabla_{x_i}}{m_3} \right],$$

(3a)

$$v^B(r,x_i) = -ia_s \frac{\lambda_1 \cdot \lambda_3}{4} \frac{1}{2m_3^2} \left[ \nabla_r \cdot \left( \frac{\sigma_3 \cdot \sigma_3}{m_3} - \frac{1}{m_3} \right) \delta(r) + \delta(r) \frac{\sigma_3 \cdot \nabla_{x_i}}{m_3} \right],$$

(3b)

for cases A and B respectively, and where $r = x_1 - x_3$. Here $a_s = g_s^2/4\pi$, and $\nabla_r$ (or $\nabla_{x_i}$) is a gradient acting on $\delta(r)$ (or the initial state wavefunction). The first parentheses in eqs. (3a,b) contain the local terms, whereas the last term is the momentum-dependent, or non-local, term.

The initial quarks as well as the final ones are taken to be states of the hamiltonian

$$H = \frac{p_i^2}{2m_i} + \frac{p_j^2}{2m_j} + \frac{\lambda_i \cdot \lambda_j}{4} \frac{k}{2} (x_i - x_j)^2,$$

(4)

where $k$ is the harmonic oscillator constant. For a meson with momentum $P$ the
The quark-antiquark (q\bar{q}) wave function is

$$\Psi(P) = \frac{e^{iP \cdot R}}{2^{3/4}} \psi_{OS} \psi_I \psi_C,$$

(5)

where \( P = p_1 + p_2, \quad R = (m_1 x_1 + m_2 x_2)/(m_1 + m_2), \) and \( \psi_I \) and \( \psi_C \) are the I-spin (flavor) and color wave functions, respectively. The q\bar{q} relative orbital-spin \( \psi_{OS} \) is

$$\psi_{OS}(\xi) = \frac{e^{-x^2/2b^2}}{(\sqrt{\pi} b)^{3/2} \chi_s}$$

(6.1)

for spatial S-states (1S) of q\bar{q}, or

$$\frac{\sqrt{\pi} \xi e^{-x^2/2b^2}}{(\sqrt{\pi} b)^{3/2} b} \sum_m (1S mm_x | J J_z) Y_{1m}(\xi) \chi_s$$

(6.2)

for spatial P-states (1P) of angular momentum J. Here \( \xi = (x_1 - x_2)/\sqrt{2}, \) \( b \) is the harmonic oscillator parameter, \( \chi_s \) is the spin of the q\bar{q} system, and \( J(J_z) \) is the total angular momentum (its magnetic quantum number) and \( Y_{1m} \) is a spherical harmonic of first order.

The relevant diagram for the decays of a q\bar{q} meson into two other mesons is shown in fig. 2. The flavor and color matrix elements can be calculated separately. In the rest frame of the initial meson the space and spin matrix elements take the form

$$\mathcal{R} = \frac{1}{2^{9/4}} \int dx_1 dx_2 dx_3 e^{-\psi_I \cdot n_1} e^{-\psi_2 \cdot n_2} \psi_{OS}^{*} \left( \frac{x_1 - x_3}{\sqrt{2}} \right) \psi_{OS}^{*} \left( \frac{x_3 - x_2}{\sqrt{2}} \right)$$

$$v(r, x_1) \psi_{OS} \left( \frac{x_1 - x_3}{\sqrt{2}} \right),$$

(7)

Fig. 2. The diagrams associated with the decay of a q\bar{q} meson into two mesons.
where \( \eta_i = (m_1 x_1 + m_3 x_3) / (m_1 + m_3) \). Performing one integration we obtain, in the rest frame of the initial meson, \( \mathcal{R} = (2\pi)^3 \delta(p_1 + p_2) \mathcal{M} \), with

\[
\mathcal{M} = 2^{3/4} \int d\xi_1 d\xi_2 \exp \left[ -\frac{i p_1 \cdot (2m_1 \xi_1 + 2m_2 \xi_2)}{\sqrt{2}} \right] \psi_{\text{OS}}(\xi_1) \psi_{\text{OS}}(\xi_2) \nonumber \]

\[
v(\xi_1, \xi_2) \psi_{\text{OS}}(\xi_1 + \xi_2),
\]  
where \( \xi_i = (x_1 - x_3)/\sqrt{2}, \xi_2 = (x_3 - x_2)/\sqrt{2} \), and \( P_i \) is the momentum of the final state meson \( i \). Here \( v(\xi_1, \xi_2) \) has the form

\[
v^A(\xi_1, \xi_2) = -i \alpha_s \frac{\lambda_1 \cdot \lambda_3}{4 m_1 m_3} \left( \frac{\sigma_3}{m_1} + \frac{\sigma_3}{m_3} - i \frac{\sigma_1 \times \sigma_3}{m_1} \right) \left( \frac{\xi_1 + \xi_2}{|\xi_1|^2} + \frac{2 \sigma_3 \cdot (\xi_1 + \xi_2)}{\sqrt{2} m_1 b^2} \right),
\]

\[
v^B(\xi_1, \xi_2) = i \frac{\alpha_s \lambda_1 \cdot \lambda_3}{2 m_1 m_3} \delta(\sqrt{2} \xi_1) \left( \frac{\sigma_3}{m_1} + \frac{\sigma_3}{m_3} - i \frac{\sigma_1 \times \sigma_3}{m_1} \right) \nonumber \]

\[
\left( -i \frac{m_1}{m_1 + m_3} P_1 - \xi_1 + \xi_2 \right) \frac{\xi_1 + \xi_2}{\sqrt{2} b^2} + \frac{\sqrt{2} \sigma_3 \cdot (\xi_1 + \xi_2)}{m_1 b^2},
\]  
respectively. Here \( b_1(b_2) \) is the harmonic oscillator parameter for a final meson and \( b \) for the initial one.

2.1. S-STATE MESON DECAYS

First we concentrate on the two-meson decays of (1S) \( qq \) states: \( \rho \rightarrow \pi \pi, \phi^* \rightarrow K \pi \) and \( \phi \rightarrow K \bar{K} \). The calculation of the matrix elements \( \mathcal{M} \) is straightforward, and the result for the space–spin part is

\[
\mathcal{M} = \alpha_s P_1 \cdot \hat{e} \Omega,
\]

where

\[
\Omega_1^{(A)} = -2^{15/4} \pi^{1/4} \frac{b_1^{1/2}}{\{2 + (b_1 / b_2)^2\}^{1/2}} \int_0^\infty du e^{-u^2} j_1 \left( \frac{(a_1 + a_2) v b_1 p_1 u}{\sqrt{2}} \right) \nonumber \]

\[
\times \left[ \left( \frac{1 - \text{Erf}(iu)}{\sqrt{\pi}} - \frac{2iv}{\sqrt{\pi}} e^{-\frac{(iv)^2}{2}} \right) \left( \frac{1}{m_1} + \frac{1}{m_3} \right) + 4r^2 \left( \frac{b_1}{b} \right)^2 \frac{u^2}{m_1} \right],
\]

\[
+ \frac{8 \nu^3}{\sqrt{\pi}} \left( \frac{b_1}{b} \right)^2 \frac{u^3}{m_1} e^{-\frac{(iv)^2}{2}} \right] \nonumber \]

\[
\Omega_1^{(B)} = -2^{-7/4} \pi^{1/4} \frac{(b/b_2)^{3/2}}{m_2^3 b_1^{3/2}} \{1 + (b_1 / b_2)^2\}^{1/2} e^{-a_2^2 p_1^2 b^2 / 4 (1 + (b_1 / b_2)^2)} \nonumber \]

\[
\times \left[ a_1 \left( 1 + \frac{b_1}{b_2} \right) \left( \frac{1}{m_1} - \frac{1}{m_3} \right) + a_2 \left( -\frac{3}{m_1} + \frac{1}{m_3} \right) \right],
\]
for case (A) and (B), respectively. Here \( \hat{e} \) is a vector-meson polarization vector, 
\[ a_i = 2m_i/(m_i + m_3), \]
\[ \nu = 1/\sqrt{1 + 2(b_1/b)^2}, \]
j_0 and \( j_1 \) are the spherical Bessel functions, and the error function is \( \text{Erf}(x) = (2/\sqrt{\pi}) \int_x^\infty e^{-t^2} dt \). Note that the term proportional to \( a_1 \) vanishes if \( m_1 = m_3 \). In order to perform the integration analytically we used a common harmonic parameter \( b_1 = b_2 \) for the final mesons 1 and 2 in eq. (11a). Also in going from eq. (8) to eq. (11a) we neglect terms proportional to \( (a_1 - a_2) \) in the exponential. This is valid exactly for \( \rho \to \pi\pi \) and \( \phi \to K\bar{K} \); and is accurate to \( \sim 10\% \) for \( K^* \to K\pi \). No such approximations are required for eq. (11b).

The decay width can be written as

\[
\Gamma = \left| \langle \lambda_1 \cdot \lambda_3 \rangle \right|^2 \left| I \right|^2 \alpha_s^2 P_i^2 \Omega_1 + \Omega_2 \right|^2 \frac{P_1 E_1 E_2}{\pi M},
\]

(12)

where the color factor is \( \langle \lambda_1 \cdot \lambda_3 \rangle = 4/(3\sqrt{3}) \), the values of the “isospin factor” are \( I = \sqrt{2} \) for \( \rho \to \pi\pi \), 1 for \( K^* \to K^0 \pi^+ \), \( 1/\sqrt{2} \) for \( K^* \to K^+ \pi^0 \), and 1 for \( \phi \to K_0 \bar{K}_0 \) and \( \phi \to K^+ K^- \). \( M \) is the mass of the decaying particle and \( E_i(\Omega_2) \) is the energy of final meson 1(2). The term \( \Omega_2 \) arises from the gluon coupling to an antiquark and is related to \( \Omega_1 \) by the interchange of \( m_1 \) and \( m_2 \).

In order to extract the effective coupling \( \alpha_s \) from experiment we must know the quark masses and the harmonic oscillator parameters. For the interaction eq. (4) the latter are given by

\[
b = (4\mu |\lambda k|)^{-1/4},
\]

(13)

where \( \lambda = \frac{4}{3} \) is the color matrix element for mesons, \( k \) is defined in eq. (4), and \( \mu \) is the reduced mass of the two interacting quarks. The above equation enables us to estimate the size parameter of the other mesons from that of the pion. We assume that the size parameter is related to the charged mean square radius via the relationship*

\[
\langle r^2 \rangle_{\text{rms}} = \frac{3}{2} b^2 m.
\]

(14)

From the measured charge radius \( \rho \) for the pion \( \sqrt{\langle r^2 \rangle} = 0.663 \pm 0.023 \) fm we obtain \( b_\rho = b_\pi = 0.766 \pm 0.027 \) fm. The experimental \( \rho \) charge radius for K is \( \sqrt{\langle r^2 \rangle} = 0.53 \pm 0.05 \) fm which gives \( b_K = 0.61 \pm 0.06 \) fm.

With such large values of \( b \), however, it is not possible to predict the mass difference of the \( \rho \) and \( \pi \) or \( K^* \) and K mesons. Furthermore, it is believed that the quark confinement radius is smaller than the rms charge radius. In this paper we leave \( b \) as a free parameter and vary it around the above values. For the up and down quark masses we use \( m_u = m_d = m = 330 \) MeV. The strange quark mass was computed from the relation \( m_s - m = m_S - m_p = 250 \) MeV which yields \( m_s = 580 \) MeV. This value is not very different from 550 MeV used by Isgur and Karl.\(^9\).

* Due to the center-of-mass corrections for \( n \) quarks in 0s harmonic oscillator orbits one finds that \( \langle r^2 \rangle = [(n - 1)/n]b^2 \).
The values of $\alpha_s$ extracted from measured decay rates are shown in Tables 1A and 1B. Here the central values of the experimental widths $\Gamma_{\text{exp}} = 153$ MeV, 51.1 MeV, 2.09 MeV and 1.45 MeV for $\rho \to \pi\pi$, $K^* \to K\pi$, $\phi \to K^+K^-$ and $\phi \to K^0\bar{K}^0$, respectively, are used. In this table the value of $\alpha_s$ extracted from the decay $\phi \to K^0\bar{K}^0$ is not shown because it is essentially the same as that extracted from the experimental rate for $\phi \to K^+K^-$. 

2.2. P-STATE MESON DECAYS

Next we evaluate the decay widths for the decay of excited (1P) q\bar{q} mesons, and in particular those for $B(1235) \to \omega \pi$, $H(1190) \to \rho \pi$, $\delta(980) \to \eta \pi$, $\epsilon(1300) \to \pi\pi$, $A_1(1270) \to \rho \pi$, $f(1270) \to \pi\pi$ and $A_2(1320) \to \rho \pi$, $\eta \pi$. The spatial–spin and isospin matrix elements can be expressed as

$$\mathcal{M} = \alpha_s G \sum_m (1S m_m) \langle \mathbf{J}_2 \rangle \{F_1(\mathbf{A})_m + F_2(\sigma_3)_m + F_3(\mathbf{C}_2(\hat{P}_1, \mathbf{A}))_m + F_4(\mathbf{C}_2(\hat{P}_1, \sigma_3))_m\},$$

$$C_2(\mathbf{X}, \mathbf{Y}) = \frac{3}{2}(\mathbf{X} \cdot \mathbf{Y})\mathbf{X} - \frac{1}{2}X^2\mathbf{Y}$$

where $\mathbf{A} = 2\sigma_3 - i\sigma_1 \times \sigma_3$ and is obtained from the local term of eq. (3), and $\hat{P}_1$ is a unit vector. The symbol $\langle \rangle$ in eq. (15) stands for the expectation values of quark spin operators for hadron states. Here we assume $m_u = m_d$. The coefficients $G$ and $F_i$ can be calculated using the following expressions:

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<th>Table 1A</th>
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<tr>
<td>$\rho \to \pi\pi$</td>
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<td>$K^* \to K\pi$</td>
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<th>Table 1B</th>
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<td>$K^* \to K\pi$</td>
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<td>$\phi \to K^+K^-$</td>
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</tbody>
</table>
\[ F_1 = B_{0,1}, \]
\[ F_2 = -3(B_{0,1} + \frac{1}{2}\nu C_{0,2}) + 4\nu^2(b_1/b)^2(B_{0,3} + \frac{1}{2}\nu C_{0,4}), \]
\[ F_3 = -2B_{2,1}, \]
\[ F_4 = -8\nu^2(b_1/b)^2(B_{2,3} + \frac{1}{2}\nu C_{2,4}), \]
\[ B_{i,n} = \int_0^\infty du u^n e^{-u^2} j_i(\sqrt{2}\nu b_1 P_1 u) \int_0^\infty dx x^2 e^{-x^2}, \]
\[ C_{i,n} = \int_0^\infty du u^n e^{-u^2} j_i(\sqrt{2}\nu b_1 P_1 u) e^{-(nu)^2}, \]
\[ (17a) \]
for case A and
\[ G^B = i \frac{\pi^{1/4} b_1^{1/2}}{2^{1/4} m^3(b_1 b_2 t)^{3/2}} e^{-P_1^2 b_1^2/4 t^2}, \]
\[ F_1^B = \left(1 - \frac{1}{t}\right) \left(1 - \frac{P_1^2 b_1^2}{6 t}\right), \]
\[ F_2^B = -2 \left(1 - \frac{1}{t} + \frac{P_1^2 b_1^2}{6 t^2}\right), \]
\[ F_3^B = -\frac{P_1^2 b_1^2}{3 t} \left(1 - \frac{1}{t}\right), \]
\[ F_4^B = -\frac{2P_1^2 b_1^2}{3 t^2}, \]
\[ t = 1 + (b_1/b_2)^2, \]
\[ (17b) \]
for case B. The wave function of the \( \omega \) meson used here is \( \omega = (u\bar{u} + d\bar{d})/\sqrt{2} \) since the \( \phi \)-meson is pure (or almost pure) \( \bar{s}s \). The \( \eta \) and \( \eta' \), on the other hand are almost pure octet and singlet, so that the isospin zero combination is \( (u\bar{u} + d\bar{d})/\sqrt{2} = \eta/\sqrt{3} + \sqrt{2}\eta'/\sqrt{3}, \) if we neglect the \( s \)-quark contribution. We use \( \eta = (u\bar{u} + d\bar{d} - 2\bar{s}s)/\sqrt{6}, \) but note that the \( \bar{s}s \) component does not contribute to the decay.

The decay rate is given by
\[ \Gamma = 2^2 |(\alpha_1 \cdot \lambda_3)|^2 |\alpha_4|^2 |G|^2 \left[ |l_1 F_1 + l_2 F_2|^2 + |l_3 F_3 + l_4 F_4|^2 \right] \frac{P_1 E_1 E_2}{\pi M}, \]
\[ (18) \]
where the factor \( 2^2 \) occurs because the gluon can couple to both quark and antiquark in the initial meson. The spin–isospin dependent coefficients \( l_1 \) to \( l_4 \) are given in
The values of $\alpha_s$ required to fit the central values of the decay widths are listed in tables 3A and 3B for cases A and B, respectively, and for various sizes of $b$ and $b_1 (= b_2)$. It is known $^{11,12}$) that the ratio of d-wave to s-wave for the final mesons is sensitive to the helicity structure of the decay. This d/s ratio in the decays $B \rightarrow \omega\pi$ and $H \rightarrow \rho\pi$ is given by

$$\frac{d}{s} = \frac{l_3 F_3 + l_4 F_4}{l_1 F_1 + l_2 F_2} = \frac{2 F_3 + F_4}{\sqrt{2} (F_1 + F_2)},$$

while d/s in $A_1 \rightarrow \rho\pi$ is given by

$$\frac{d}{s} = \frac{l_3 F_3 + l_4 F_4}{l_1 F_1 + l_2 F_2} = \frac{F_3 + F_4}{2 \sqrt{2} (F_1 + F_2)}.$$

| Table 2 | The values of coefficients $l_1$, $l_2$, $l_3$ and $l_4$ |
|---|---|---|---|---|---|---|---|
| Particle | $I^G(J^P)C_n$ | Mode | Partial width (MeV) | $l_1$ | $l_2$ | $l_3$ | $l_4$ |
| B(1235) | $1^+(1^+)$ | $\omega\pi$ | $150 \pm 10$ | $-2$ | $-1$ | $-\sqrt{2}$ | $-1/\sqrt{2}$ |
| H(1190) | $0^+(1^+)$ | $\rho\pi$ | $320 \pm 50$ | $-2\sqrt{3}$ | $-\sqrt{3}$ | $-6$ | $-\sqrt{6}$ |
| $\delta$ (980) | $1^-(0^+)$ | $\eta\pi$ | $54 \pm 7$ | $0$ | $1$ | $0$ | $0$ |
| $\epsilon$ (1300) | $0^+(0^+)$ | $\pi\pi$ | $150-400$ | $0$ | $3/\sqrt{2}$ | $0$ | $0$ |
| $A_1$ (1270) | $1^-(1^+)$ | $\rho\pi$ | $316 \pm 45$ | $2$ | $2$ | $-1/\sqrt{2}$ | $-1/\sqrt{2}$ |
| $f$ (1270) | $0^+(2^+)$ | $\pi\pi$ | $176 \pm 20$ | $0$ | $0$ | $0$ | $-3/2\sqrt{5}$ |
| $A_2$ (1320) | $1^-(2^+)$ | $\rho\pi$ | $77 \pm 4$ | $0$ | $0$ | $-3/\sqrt{10}$ | $-3/\sqrt{10}$ |

| Table 3A | The table of $\alpha_s$ and d/s for the case A interaction |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Particle | Mode | $b = 0.4$ | $b_1 = b_2 = 0.4$ |
| | | $0.4$ | $0.4$ | $0.6$ | $0.6$ | $0.6$ | $0.6$ | $0.8$ | $0.8$ | $0.8$ | $0.8$ | $0.8$ | $0.8$ |
| B | $\omega\pi$ | 0.66 | 1.0 | 1.3 | 0.67 | 0.71 | 0.80 | 0.85 | 0.74 | 0.72 |
| H | $\rho\pi$ | 0.62 | 0.90 | 1.2 | 0.60 | 0.66 | 0.76 | 0.76 | 0.67 | 0.67 |
| $\delta$ | $\eta\pi$ | 0.29 | 0.47 | 0.75 | 0.30 | 0.35 | 0.46 | 0.38 | 0.36 | 0.41 |
| $\epsilon$ | $\pi\pi$ | 0.22 | 0.31 | 0.41 | 0.26 | 0.29 | 0.34 | 0.38 | 0.37 | 0.40 |
| $A_1$ | $\rho\pi$ | 0.34 | 0.53 | 0.76 | 0.36 | 0.40 | 0.49 | 0.47 | 0.43 | 0.45 |
| $f$ | $\pi\pi$ | 1.2 | 0.79 | 0.77 | 1.8 | 0.86 | 0.66 | 3.6 | 1.3 | 0.86 |
| $A_2$ | $\eta\pi$ | 1.1 | 0.68 | 0.65 | 1.6 | 0.69 | 0.52 | 3.0 | 0.97 | 0.61 |
| $A_2$ | $\rho\pi$ | 1.2 | 0.83 | 0.80 | 1.6 | 0.73 | 0.56 | 2.6 | 0.90 | 0.58 |
| $B \rightarrow \omega\pi$ | d/s | $0.21$ | $0.45$ | $0.70$ | $0.17$ | $0.40$ | $0.63$ | $0.15$ | $0.36$ | $0.60$ |
| $H \rightarrow \rho\pi$ | d/s | $0.19$ | $0.41$ | $0.62$ | $0.15$ | $0.36$ | $0.58$ | $0.13$ | $0.32$ | $0.54$ |
| $A_1 \rightarrow \rho\pi$ | d/s | $0.71$ | $1.6$ | $2.4$ | $0.55$ | $1.4$ | $2.2$ | $0.44$ | $1.2$ | $2.0$ |

We use central values of experimental width $^{10}$) to extract the values of $\alpha_s$. 
The above equations are sometimes written as \( \frac{d}{s} = \frac{-\sqrt{2}(g_0-g_1)}{(g_0+2g_1)} \), where \( g_1 \) (\( g_0 \)) is the vector meson helicity amplitude with helicity one (zero).

3. Discussion

In table 1 we present the values of \( \alpha_s \) required in order to fit the decay widths of the S-state mesons for various values of the harmonic oscillator parameters. The value of \( b \) is that for the initial meson, whereas \( b_1 \) and \( b_2 \) are those for the final state mesons. It can be seen that the values of \( \alpha_s \) are relatively independent of these parameters for case A (Coulomb-type propagator) as long as \( b \) and \( b_1 \) or \( b_2 \) do not differ too much from each other. For case B (constant propagator), \( \alpha_s \) increases with confinement radius. This increase in \( \alpha_s \) is required to compensate for the smaller energy released in the decays and indicates that this approximation may not be as suitable as taking for the propagator the value of the energy released in the reactions. Furthermore, it is seen that case B leads to unreasonably large values of \( \alpha_s \) when \( b \) becomes large.

It is not clear to us, a priori, which form of the propagator is more desirable. For nucleons at rest, where the confinement kinetic energies of the quarks are small (e.g., for up and down quarks of masses 330 MeV and a nucleon of mass 938 MeV), the momentum of the confined quarks may be small. But for pions where the confinement energy is large, or for \( \rho \) and \( \omega \) mesons where the masses are considerably larger than those of the quark constituents, this is much less clear. It is likely that both momentum and energy are being transferred by the gluon. For simplicity we neglect one of these, and approximate the energy transferred by the masses of the quarks being created by the gluon. But one can question either approximation, and since the propagating gluon is also confined, one can question the use of a free
gluon propagator. The use of a constant quark propagator may be sensible, but it may not be appropriate to use $2m_q$ for this constant. The value of $\alpha_s$ for case B scales approximately as $m^2$, i.e. the squared effective mass of the propagator, so that the use of a smaller effective mass for the propagator would substantially reduce the values of $\alpha_s$ found in table 1.

In all the examples shown in table 1 the values of $\alpha_s$ found decrease somewhat for the decays of more massive mesons. However, it should be noted that the errors in the decay widths of the various mesons are not reflected in the values of $\alpha_s$ listed in the table. Although the momentum transfer does not necessarily increase, this small decrease in $\alpha_s$ is not unreasonable. In particular, it may simply indicate that the decay processes occur at shorter distances, which may be sensible, since the Compton wavelengths are smaller for the more massive mesons. The fact that the values of $\alpha_s$ found in tables 1A or 1B for particular values of $b$ and $b_1 = b_2$ are consistent with each other may follow from the SU(3) relation between the decaying mesons, but this consistency is absent in tables 3A and 3B.

The upper parts of 3A and 3B are similar to tables 1A and 1B, except that they are for the P-state mesons. We would hope that the values of $\alpha_s$ deduced from the decay widths of these mesons would be similar to those of the vector mesons in table 1, but such is not the case, in general. Mostly, the problem seems to occur for the smaller radii for case A and for all radii for case B. Thus, we believe that the variations of $\alpha_s$ for different mesons are excessive for both tables. There is no choice of radii for which tables 1A and 3A are consistent with each other for all mesons. However, as in table 1, it is found that the values of $\alpha_s$ in table 3 are less sensitive to the meson radii for case A than B. The values of $\alpha_s$ required for the decays of the $f$ and $A_2$ mesons, for which only final state $d$-waves contribute, are close to those of (quark) S-state mesons, e.g., $\rho$, $\omega$ mesons. The $\delta$ and $\epsilon$ mesons, where only final state $s$-waves contribute, require considerably smaller values of $\alpha_s$ to fit the decay rates. Intermediate values of $\alpha_s$ match the measured decay rates of the B, H, and $A_1$ mesons, for which both $s$- and $d$-waves contribute. All of these results are tempered by the large experimental errors for the widths of some of the decays (e.g., that of $\pi^+ \to \pi\pi$). There is no parameter to distinguish $s$-wave from $d$-wave decays, unlike the model of Godfrey and Isgur 14).

In the case of the IP mesons it is also of interest to obtain the helicity structure of the decays. This can be done when one of the final state particles is a vector meson. The helicity structure can be related to the polarization of the vector meson or the ratio of $d$- and $s$-orbital angular momenta produced. Experimentally, for the B-meson, the ratio of $d$- to $s$-state is $0.29 \pm 0.05$ [ref. 10)]. By contrast, we obtain a negative value for this ratio. For case A it is dependent on meson radii, but for case B it is independent of this ratio and is equal to $-\sqrt{2}$. This constant value can be understood by examining eq. (19). In this case $2F_3 + F_4 = 2(2F_1 + F_2)$ and $g_1 = 0$ because of the cancellation between the local and nonlocal terms in eq. (3b); it is then readily seen that $d/s = -\sqrt{2}$. (This is not true for the $A_1 \to \rho\pi$ decay.) If we
neglect the momentum-dependent (nonlocal) term in eq. (3b), then the d/s ratio for the B-meson decay is 0.09, 0.15, and 0.21 for $b = b_1 = b_2 = 0.5 \text{ fm}$, 0.6 fm, and 0.7 fm, respectively. These results are clearly in better agreement with experiment. However, the same neglect of the nonlocal or velocity-dependent terms would give no $\rho$ decay because equal quark masses $m_u = m_d$ are assumed. In $\rho$ decay the expectation values of the spin operators in eq. (9) are related by $2<\sigma_3> = -(-i\sigma_1 \times \sigma_3)$, so that the contribution from the local term vanishes. Hence this neglect cannot be the answer to our problem. It should be noted that the d/s ratio is also negative for case A; however, here it varies with radius due to the spatial dependence of the propagator. There is still a cancellation between the local and nonlocal terms of eq. (3a), but the cancellation is no longer exact.

We believe that it is this partial cancellation which may be responsible for the variations in $\alpha_s$ required to fit the decay widths, as well as for our inability to fit the d/s ratio of the B-meson decay. This partial cancellation does not occur for $p\bar{p}$ annihilation. In particular, the cancellation here means that the decays of the mesons that we have considered are good tests of quark models since they are sensitive to details. It may well be that other types of diagrams than those we consider are required, e.g., higher order gluonic terms. However, it may also just mean that, in our model, relativistic corrections are more important in the decays of the mesons than for other processes where the cancellation is absent. To investigate this suggestion, we examine higher order (i.e. relativistic) terms in the momentum of the quarks on the d/s ratio.

In this case the terms in the square brackets of eq. (2) change to

\[
\left(1 - \frac{p_1^2 + p_2^2 + p_3^2 + p_5^2}{8m^2}\right) \frac{2\sigma_3 \cdot q - 2\sigma_3 \cdot p_1 - i\sigma_1 \times \sigma_3 \cdot q}{2m} + \frac{p_1 \cdot p_3 \sigma_3 \cdot q - p_3^2 \sigma_3 \cdot p_3 - p_5^2 \sigma_3 \cdot p_3 + p_7^2 \sigma_3 \cdot p_7}{8m^3} \frac{i\sigma_3 \cdot (q \times p_1)\sigma_3 \cdot q}{8m^3} \frac{\sigma_3 \cdot p_3 \sigma_3 \cdot (2p_1 - q)\sigma_3 \cdot p_2}{4m^3} + \frac{\sigma_3 \cdot p_3 (i\sigma_1 \times \sigma_3) \cdot p_3}{4m^3},
\]

where $m_1$ is set equal to $m$. After integration over the internal variables, the decay amplitude, which corresponds to eq. (15), has the form

\[
\mathcal{M} \sim \langle \sigma_3 \rangle m \left\{ -\frac{P_1^2 b_2^2}{18} + \frac{35}{2304} \frac{P_1^4 b_2^2}{m^2} - \frac{19}{288} \frac{P_1^2}{m^2} + \frac{5}{48m^2 b_2^2} \right\} + \langle C_2 (\hat{P}_1, \sigma_3) \rangle m \left\{ \frac{P_1^2 b_2^2}{9} + \frac{35}{1152} \frac{P_1^4 b_2^2}{m^2} - \frac{11}{288} \frac{P_1^2}{m^2} \right\},
\]

(22)
for case B, where $b = b_1 = b_2$ is used for simplicity. The first terms in each brace are those of lowest order in quark momenta. The numerical values of $d/s$ for various values of $b$ and case B are 0.14 ($b = 0.3$ fm), 1.6 ($b = 0.5$) and -2.0 ($b = 0.7$), respectively. The strong $b$ dependence is mainly due to the cancellations among the terms.

In any case, it appears that inclusion of relativistic corrections may lead to a fit of the $d/s$ ratio or the helicity structure of the decay of the B meson. It would be interesting to carry out a fully relativistic calculation\(^{15}\) of our model to ascertain the correctness of this suggestion; this is not done here.

In conclusion, we have examined the decays of the low lying mesons [quark S-state mesons and excited (P-state) mesons] in a one colored vector boson exchange or an effective perturbative QCD approach. We find that for either a Coulomb-like gluon propagator or for a constant ($= 2m_q$ for the energy transfer) propagator we are not able to fit the decays of the excited mesons with a single value or a small range of $\alpha_s$. In general, the values of $\alpha_s$ required for the excited mesons tend to be smaller than those for the S-state (low-lying vector) mesons. We are also unable to fit the sign of the $d/s$ ratio for the B meson decay. We believe that part or all of the problem arises from the exact or near cancellation of the local and nonlocal terms in the interaction responsible for the decay amplitude. We have shown that, due to this cancellation, relativistic corrections are important and of the right sign to help obtain better agreement with the experimentally deduced helicity structure.

We acknowledge that the so-called $^{3}P_0$ model\(^{12}\) is better able to reproduce experimental data. Lattice calculations\(^{16}\) and flux-tube models\(^{17}\) appear to give results closer to this formulation than to ours.

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