Two-photon decay of the Higgs boson \( H \rightarrow \gamma\gamma \) and its reverse process \( \gamma\gamma \rightarrow H \) are important processes in detecting Higgs boson which plays a fundamental role in the Standard Model of electroweak interaction. We calculate the QCD corrections to these processes for all mass ranges of the Higgs boson without any constraint for the top quark mass. For a heavy Higgs boson \( (m_H \gg m_t) \), the QCD corrections become very large \((50-100\%)\), while for a lighter one \( (m_H \lesssim m_t) \), the corrections are negligible.

The Standard Model of electroweak interaction\(^1\) made a remarkable success in describing many experimental results. However, two important ingredients of the model, the top quark and the Higgs boson, have not been observed. Especially the latter which is a direct consequence of electroweak symmetry breaking and the detection of Higgs boson is a crucial test for Standard Model. In the minimal Standard Model, the Higgs boson with a mass below 90 GeV has already been excluded experimentally.\(^2\) The Higgs boson will be found in the experiments of LEP200 if its mass is below 100 GeV. If its mass is heavier than 100 GeV, we have to wait for next TeV-energy colliders, hadron colliders like LHC or \( e^+e^- \) linear colliders.

In high energy hadron colliders, the Higgs boson will be produced dominantly from the gluon-gluon fusion. For the intermediate mass Higgs boson \( (m_W < m_H < 2m_Z) \), \( H \rightarrow b\bar{b} \) is the major decay mode. But it is very difficult to detect this signal because of the large QCD background. But \( H \rightarrow \gamma\gamma \) is promising in spite of the small branching ratio \( O(10^{-3}) \). On the other hand, the Higgs boson production in high energy photon-photon collisions has been proposed recently.\(^3\) For the heavy
Higgs boson \((m_H > 2m_Z)\), feasible processes are \(\gamma\gamma \to H \to ZZ, W^+W^-, t\bar{t}\) and \(b\bar{b}\). The attractive process is \(\gamma\gamma \to H \to ZZ\), owing to the absence of a tree-level background to this process.\(^4\) We calculate the second order QCD corrections to the decay width of \(H \to \gamma\gamma\). Using this decay width, we can immediately obtain the scattering cross-section for \(\gamma\gamma \to H \to ZZ\) at the peak of the Higgs boson:

\[
\sigma(s) = \frac{8\pi \Gamma_{H \to \gamma\gamma} \Gamma_{H \to ZZ}}{(s - m_H^2)^2 + \Gamma_H^2 m_H^2}. \tag{1}
\]

![Typical one-loop Feynman diagrams for \(H \to \gamma\gamma\).](image)

At one-loop level, the Higgs boson decays into two photons through fermion loops and \(W\) boson loops (Fig. 1). The decay width is expressed by the fermion contribution \(I_f\) and the \(W\) boson contribution\(^5\) \(I_W\):

\[
\Gamma(H \to \gamma\gamma) = \frac{G_F a^2 m_Z^2}{8\sqrt{2}\pi^3} \left| \sum_f N_c Q_f^2 I_f(\lambda_f) + I_W(\lambda_W) \right|^2, \tag{2}
\]

where \(N_c\) is the color factor, \(Q_f\) the fermion charge in units of \(e\) and

\[
\lambda_f = \frac{m_f^2}{m_H^2}, \quad \lambda_W = \frac{m_W^2}{m_H^2}. \tag{3}
\]

Here, the functions \(I_f\) and \(I_W\) are given by

\[
I_f(\lambda_f) = 2\lambda_f + \lambda_f(4\lambda_f - 1)f(\lambda_f),
\]

\[
I_W(\lambda_W) = -\frac{1}{2} - 3\lambda_W + 3\lambda_W(1 - 2\lambda_W)f(\lambda_W), \tag{4}
\]

with

\[
f(\lambda) = \begin{cases} 
\frac{1}{2} \left( \ln \frac{\eta_-}{\eta_+} + i\pi \right)^2 & \text{for } 0 < \lambda < \frac{1}{4} \\
-2 \arcsin^2 \left( \frac{1}{2\sqrt{\lambda}} \right) & \text{for } \lambda > \frac{1}{4},
\end{cases} \tag{5}
\]

where

\[
\eta_{\pm} = \frac{1}{2} \left( 1 \pm \sqrt{1 - 4\lambda} \right). \tag{6}
\]
The value of $I_f(\lambda_f)$ becomes small for small $\lambda_f$. This implies that the top quark contribution is dominant in all fermions. When we consider beyond the Standard Model such as SUSY GUT and technicolor, extra charged particles which couple to the Higgs boson also contribute to the decay width. Because the coupling constant is proportional to the particle mass, decoupling theorem does not hold and the decay width is sensitive to new heavy particles which appear beyond the Standard Model.

QCD corrections to $H \to \gamma\gamma$ were first calculated by Zheng and Wu in the limit of $m_t/m_H \to \infty$. Djouadi, Spira, van der Bij and Zerwas gave numerical result above threshold ($m_H < 2m_W$). We employ dispersion relation technique which enables us to calculate the decay width for all Higgs mass region.

From the gauge invariance, the amplitude of the $H \to \gamma\gamma$ decay has the form

$$A(t)[-(p_1p_2)g_{\mu\nu} + p_{\mu}^2p_{\nu}^2],$$

where $p_1$ and $p_2$ are photon momenta and $t = (p_1 + p_2)^2$. We obtain the imaginary part of $A(t)$ from cut graphs in which intermediate states are on-shell. Using the dispersion relation, the real part of $A(t)$ is calculated numerically:

$$\text{Re} A(t) = \frac{1}{\pi} P \int_0^\infty \frac{\text{Im} A(t')}{t' - t} dt',$$

where $P$ stands for principal value of the integral. We adopt the on-shell renormalization scheme and introduce a regularizing photon mass $\lambda$. We calculate both two-body cut contributions (Fig. 2) and three-body cut contributions (Fig. 3). The top quark contribution to the decay width $I_t(\lambda_t)$ is modified to

$$I_t^{\text{tot}}(\lambda_t) = I_t(\lambda_t) + I_t^{\text{QCD}}(\lambda_t)$$

and $I_t^{\text{QCD}}(\lambda_t)$ becomes

$$\text{Im} I_t^{\text{QCD}}(\lambda_t) = \alpha_s C_F \frac{x}{(1 + x)^6} \left[(1 - x^4) \left\{ -16(\text{Li}_3(x) + \text{Li}_3(-x)) + \log x \left(4 \text{Li}_2(x) + 7 \text{Li}_2(-x)\right) + \frac{1}{12} \log^2 x + \frac{3}{2} \zeta(2) \log x + 4\zeta(3) \right\} + (1 + x)^2 \left\{ -4(1 - x)^2 \text{Li}_2(x) - 2(3 + 2x + 3x^2) \text{Li}_2(-x) - (5 + 6x + 5x^2) \log x \log(1 + x) + 6x \log x + 3(1 - x^2) \right\} + \frac{1}{4} (1 + x)(-3 + 25x + 7x^2 + 3x^3) \log^2 x + \frac{1}{2} (1 + x)(5 - 27x - 9x^2 - x^3) \zeta(2) \right\}.$$
Fig. 2. Two-body cut contribution to the QCD corrections of $H \to \gamma\gamma$.

![Diagram](a) ![Diagram](b1) ![Diagram](b2)

![Diagram](c) ![Diagram](d)

Fig. 3. Three-body cut contribution to the QCD corrections of $H \to \gamma\gamma$.

![Diagram](a1) ![Diagram](a2) ![Diagram](b1)

![Diagram](b2) ![Diagram](c1) ![Diagram](c2)

where

\[ z = \frac{1 - \sqrt{1 - 4m_t^2/t}}{1 + \sqrt{1 - 4m_t^2/t}}, \]

\[ \text{Li}_2(x) = -\int_0^x \frac{\log(1 - t)}{t} \, dt, \]

\[ \text{Li}_3(x) = \int_0^x \frac{\text{Li}_2(t)}{t} \, dt, \]

\[ \zeta(s) = \sum_{n=0}^{\infty} \frac{1}{n^s}. \]

To calculate the box diagram of Fig. 2c, we employ Brown-Feynman method which reduces loop-momentum integrals to the sum of scalar integrals with numerator
being one. Infrared divergence is canceled out between two-body cut contributions and three-body cut contributions. Real and imaginary parts of $I_t(\lambda_t)$ and $I_t^{\text{QCD}}(\lambda_t)$ are given in Fig. 4 for $m_t = 160$ GeV. The decay width of $H \rightarrow \gamma\gamma$ process is shown in Fig. 5. QCD corrections reduce the decay width in the Higgs mass range

\[ \alpha_s = \alpha_s(m_H^2) \text{ for } m_H > 2m_t, \quad \alpha_s(4m_t^2) \text{ for } m_H < 2m_t. \]
320 GeV to 550 GeV. Above 550 GeV, they increase the decay width again and the width becomes twice as large as the lowest process. Such large QCD corrections also appear in the Higgs boson production in $\gamma\gamma$ collision, so QCD corrections play very important roles in $\gamma\gamma \rightarrow H \rightarrow ZZ$.

Note Added

After completing this work, one of the authors (R.N.) was informed by I. F. Ginzburg that similar calculation has been done by K. Melnikov and O. Yakovlev (Phys. Lett. B312, 179 (1993)). But they did not give analytical expression for the amplitude or the decay width.

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