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Maximum Likelihood Estimates of Parameters in Various Types of Distribution Fitted to Important Data Cases

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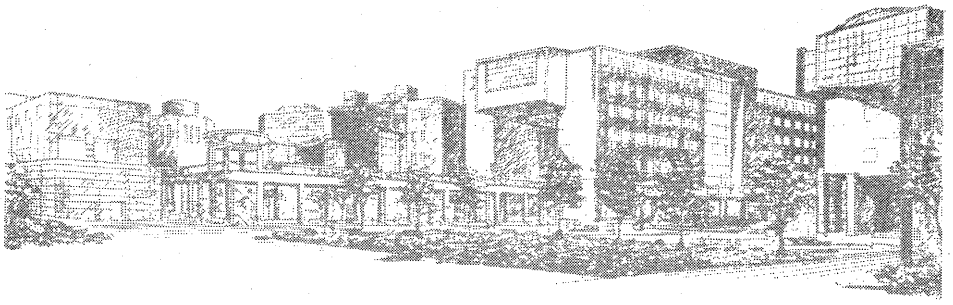

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Abstract

Maximum likelihood estimates of parameters in various types of distribution fitted to important data cases are listed.

TYPES OF THE DISTRIBUTION:

- Normal distribution (2-parameter)
- Uniform distribution (2-parameter)
- Exponential distribution (2-parameter)
- Weibull distribution (2-parameter)
- Gumbel Distribution (2-parameter)
- Weibull/Frechet Distribution (3-parameter)
- Generalized extreme-value distribution (3-parameter)
- Gamma distribution (3-parameter)
- Extended Gamma distribution (3-parameter)
- Log-normal distribution (3-parameter)
- Extended Log-normal distribution (3-parameter)
- Generalized gamma distribution (3-parameter)
- Generalized gamma distribution (4-parameter)
- Extended Generalized gamma distribution (4-parameter)
- Extended Weibull distribution (3-parameter)
- Extended Exponential distribution (2-parameter)
- Log-gamma distribution (log-Pearson type III) (3-parameter)

Table 1. Data (#1 - 13)

data case	data									
1	2.1	3.4	3.7	2.7	3.2	3.5	2.7	2.7	3.4	3.3
	3.0	2.9	3.0	3.4	2.8	3.5	3.5	3.4	3.4	3.6
2	3.4	3.8	3.1	2.6	2.9	3.0	3.2	3.5	3.5	3.7
	3.1	3.6	3.5	2.8	2.9	3.1	2.9	2.9	2.7	3.6
3	3.3	2.5	3.0	3.0	3.2	2.5	3.0	2.9	3.5	3.6
	2.8	3.8	2.4	3.0	3.4	2.5	3.3	3.5	3.8	3.6
4	3.0	4.2	3.8	3.8	3.3	2.7	2.7	3.3	3.2	3.5
	3.5	2.6	3.1	3.4	2.8	2.9	3.1	3.1	3.6	3.4
5	3.2	2.7	3.4	3.8	3.4	3.9	2.6	3.1	3.6	3.3
	2.7	2.9	3.2	2.5	3.1	2.4	2.6	3.3	2.8	3.8
6	.654	.613	.315	.449	.297	.402	.379	.423	.379	.324
	.269	.740	.418	.412	.494	.416	.338	.392	.484	.265
7	1364	2154	2236	2518	2527	2600	3009	3045	4109	5500
	5800	7200	8400	8400	8900	11500	12700	15300	18300	20400
8	184	250	439	444	450	478	487	524	688	850
	1048	1280	1364	1488	1513	1860	1947	1991	2200	2446
9	.55	.74	.77	.81	.84	.93	1.04	1.11	1.13	1.24
	1.25	1.27	1.28	1.29	1.30	1.36	1.39	1.42	1.48	1.48
	1.49	1.49	1.50	1.50	1.51	1.52	1.53	1.54	1.55	1.55
	1.58	1.59	1.60	1.61	1.61	1.61	1.61	1.62	1.62	1.63
	1.64	1.66	1.66	1.66	1.67	1.68	1.68	1.69	1.70	1.70
	1.73	1.76	1.76	1.77	1.78	1.81	1.82	1.84	1.84	1.89
	2.00	2.01	2.24							
10	.37	.40	.70	.75	.80	.81	.83	.86	.92	.92
	.94	.95	.98	1.03	1.06	1.06	1.08	1.09	1.10	1.10
	1.13	1.14	1.15	1.17	1.20	1.20	1.21	1.22	1.25	1.28
	1.28	1.29	1.29	1.30	1.35	1.35	1.37	1.37	1.38	1.40
	1.40	1.42	1.43	1.51	1.53	1.61				
11	2398	2812	3113	3212	3523	5236	6215	6278	7725	8604
	9003	9350	9460	11584	11825	12628	12888	13431	14266	17809
12	35	35	31	29	28	25	24	24	23	23
	22	20	20	19	19	18	16	16	13	12
	12	12								
13	152.7	172.0	172.5	173.3	193.0	204.7	216.5	234.9	262.6	422.6

Table 1. Data (#14 - 21)

data case	data									
14	15	20	27	42	42	43	44	46	64	65
	65	68	68	71	74	75	75	76	77	78
	92	95	100	102	102	112	113	116	117	124
	124	126	127	134	149	152	153	161	168	205
15	40.9	52.2	53.2	59.4	60.0	66.8	77.3	78.0	79.7	81.1
	81.4	85.4	86.0	86.3	87.4	88.5	89.9	92.4	93.0	93.2
	108.7	109.3	111.6	113.1	114.2	117.7	121.6	121.9	127.6	128.0
	129.7	130.8	134.1	137.5	139.2	140.3	153.0	153.8	183.3	185.1
16	3.1	4.6	5.6	6.8						
17	200	6091	336	327	154	109	111	282	2120	1082
	918	718	482	1345	53600	5900	1918	900	1045	1415
18	148.290		144.328		174.800		168.554		184.101	
	166.475		131.375		145.788		135.880		137.338	
	164.304		155.369		127.211		132.971		128.709	
	201.415		133.143		155.680		153.070		157.238	
19	1×2									
	2×6									
	3×17									
	4×77									
	5×96									
	6×73									
	7×22									
	8×8									
	9×3									
	10×3									
	12×1									
	13×1									
	19×1									
20	1.5	1.7	1.8	2.0	2.2	2.2	2.3	2.5	2.5	2.5
	2.7	2.9	2.9	3.1	3.1	3.1	3.1	3.2	3.3	3.3
	3.6	3.9	4.0	4.1	4.1	4.2	4.2	4.5	4.6	4.8
	4.9	5.2	5.6	5.7	5.9	6.4	6.6	6.9	6.9	7.4
	7.5	8.0	9.2	9.5	9.6	10.3	11.6	20.3		
21	370	392	836	477	488	335	752	389	566	364
	414	263	662	281	295	363	418	216	190	489
	434	362	374	339	375	753	330	571	258	579
	393	569	310	177	188	230	395	204	236	475
	484	333	301	425	487	708	459	154	565	648
	273	441	489	1105	618	433	605	652	239	330
	445	645	301	337	391	192	403	485	423	325
	302	420	324	468	441	342	780	280	283	637
	316	322	890	275	157	653	380			

Table 1. Data (#22 - 26)

data case	data									
22	17.88 55.56 127.92	28.92 67.80 128.04	33.00 68.64 173.40	41.52 68.64	42.12 68.88	45.60 84.12	48.48 93.12	51.84 98.64	51.96 105.12	54.12 105.84
23	216 1438	245 1698	350 1873	632 1967	864 2359	1187 2628	1216 2877	1292 3348	1355 3542	1417 4952
24	$e^{-0.216}$ $e^{4.054}$ $e^{0.438}$ $e^{1.954}$		$e^{-0.410}$ $e^{0.032}$ $e^{2.208}$ $e^{0.716}$		$e^{-1.064}$ $e^{1.262}$ $e^{-0.754}$ $e^{-1.686}$		$e^{-6.824}$ $e^{-0.030}$ $e^{-0.684}$ $e^{-0.832}$		$e^{-0.758}$ $e^{-2.640}$ $e^{-3.506}$ $e^{-0.438}$	
25	8.75	11.15	13.44	15.80	18.33	21.16	24.45	28.53	34.14	44.06
26	17.8 36.6	21.3 38.5	23.8 39.7	25.9 41.2	27.4 43.4	29.4 44.5	30.6 47.0	32.3 48.8	33.5 52.5	34.9 61.4
27	565 524 518 394 371 190 564 255	294 368 365 348 824 202 479 292	303 464 515 428 292 405 303	569 411 280 337 345 583 603	232 368 289 311 442 725 514	405 487 255 453 360 232 377	228 394 334 328 371 974 318	232 337 456 564 544 456 342	394 385 479 527 552 289 593	238 351 334 510 651 348 378

Table 2. MLE's for the Normal Distribution

data case	estimates		
	$\hat{\sigma}$	$\hat{\mu}$	$\log L_{\max}$
1	.396737	3.16000	−9.88912
2	.349142	3.19000	−7.33323
3	.430232	3.13000	−11.5102
4	.405586	3.25000	−10.3303
5	.444157	3.11500	−12.1472
6	.122107	.423125	13.6784
7	5548.38	7298.1	−200.804
8	696.931	1096.55	−159.313
9	.321543	1.50683	−17.9118
10	.268409	1.13000	−4.77001
11	4333.88	8568.00	−195.863
12	6.71956	21.6545	−73.1271
13	74.3821	220.480	−57.2815
14	43.2836	92.6750	−207.468
15	33.7523	104.815	−197.519
16	1.35716	5.02500	−6.89732
17	11509.6	3952.65	−215.398
18	19.4484	152.302	−87.7341
19	1.68904	5.19355	−602.361
20	3.31788	5.02917	−125.677
21	176.694	426.184	−573.622
22	36.6647	72.2243	−115.477
23	1188.98	1772.80	−169.996
24	12.4258	4.42219	−78.7742
25	10.4968	21.9810	−37.7001
26	10.8099	36.5250	−75.9881
27	142.757	413.362	−459.366

$$f(x;\sigma,\mu) = \frac{1}{\sqrt{2\pi}\sigma}\exp\left\{-\frac{(x-\mu)^2}{2\sigma^2}\right\}$$

Table 3. MLE's for the Two−parameter Weibull Distribution

data case	estimates		
	$\hat{\eta}$	$\hat{\beta}$	$\log L_{\max}$
1	3.32390	10.6107	−8.18392
2	3.34870	10.1456	−7.80851
3	3.31666	8.48388	−11.3481
4	3.43280	8.23286	−11.5332
5	3.31012	7.73754	−12.5830
6	.468862	3.52595	13.2640
7	8044.79	1.38451	−196.375
8	1227.89	1.61705	−157.082
9	1.62811	5.78070	−15.2068
10	1.22965	5.14748	−3.34939
11	9697.47	2.10598	−194.916
12	24.0921	3.50672	−73.0153
13	246.409	2.93592	−57.3013
14	104.638	2.27436	−206.270
15	116.781	3.36525	−197.323
16	5.53935	4.31300	−6.82431
17	1917.24	.571971	−175.538
18	161.186	7.65769	−89.5248
19	5.75955	2.87330	−613.510
20	5.68295	1.68137	−115.860
21	480.750	2.53383	−570.278
22	81.8783	2.10206	−113.691
23	1965.83	1.51252	−167.337
24	1.84345	.505731	−35.7180
25	24.9360	2.25276	−37.0484
26	40.5008	3.66281	−75.9982
27	462.150	2.95680	−459.135

$$F(x;\eta,\beta) = 1 - \exp[-(x/\eta)^{\beta}], \quad (\eta > 0,\ \beta > 0,\ x \geq 0).$$

Table 4. MLE's for the Two−parameter Gumbel Distribution

data case	estimates		
	$\hat{\eta}$	$\hat{\gamma}$	$\log L_{\max}$
1	.294495	3.33961	−7.74824
2	.323376	3.36586	−8.29778
3	.377824	3.34074	−11.6892
4	.424575	3.45947	−12.7342
5	.420226	3.3392	−13.3312
6	.143354	.489785	9.54874
7	6257.87	10302.1	−204.433
8	688.366	1456.63	−161.148
9	.271875	1.65359	−14.9561
10	.218517	1.25414	−2.17102
11	4267.70	10770.7	−197.499
12	6.82062	25.1235	−75.4281
13	97.8594	263.311	−60.2121
14	45.2295	114.97	−212.187
15	35.2968	122.216	−202.272
16	1.17697	5.69034	−6.91296
17	19727.7	11347.5	−225.292
18	21.7302	162.649	−91.0974
19	3.20861	6.19360	−768.029
20	5.11853	6.98107	−144.682
21	222.639	522.514	−594.926
22	42.7939	92.0232	−120.038
23	1379.96	2413.81	−173.887
24	21.1547	12.3749	−88.5559
25	11.3465	27.5571	−39.2035
26	11.2015	42.0918	−78.2604
27	186.659	491.897	−797.408

$$F(x;\eta,\gamma) = 1 - \exp(-\exp\{(x-\gamma)/\eta\})$$

Table 5. MLE's for the Three–parameter Weibull/Frechet Distribution

data case	estimates			
	$\hat{\eta}$	$\hat{\beta}$	$\hat{\gamma}$	$\log L_{\max}$
1	1.71343	−6.27602	5.07723	−7.49358
2	.793441	2.12879	2.48764	−6.66366
3	1.44103	3.4386	1.83887	−11.1484
4	.835128	1.87733	2.50704	−9.45101
5	.929572	1.9177	2.28934	−11.3767
6	.172736	1.24447	.261145	17.0683
7	5934.10	1	1364.00	−193.769
8	964.410	1.15229	174.270	−156.264
9	3.23502	11.8558	−1.5934	−14.2853
10	4.72048	21.3121	−3.4719	−2.08236
11	6736.97	1.25556	2241.04	−194.404
12	11.7528	1.51553	10.9878	−71.7569
13	67.7800	1	152.700	−52.1627
14	101.795	2.19942	2.47867	−206.258
15	83.5086	2.32998	30.7992	−196.518
16	1.92500	1	3.10000	−6.61970
17	3843.65	1	109.000	−185.084
18	26.3172	1.13042	126.985	−84.4232
19	4.83328	2.49067	.864659	−603.005
20	3.69115	1.11873	1.48840	−108.198
21	315.939	1.64258	143.296	−564.101
22	63.8810	1.59430	14.8759	−112.850
23	1656.55	1.16740	191.333	−166.998
24	4.42110	1	.00108736	−49.7278
25	13.2310	1	8.75000	−35.8256
26	25.0780	2.16520	14.3023	−75.3666
27	258.111	1.66826	182.649	−451.383

$$F(x;\eta,\beta,\gamma) = 1 - \exp\left\{-\left(\frac{x-\gamma}{\eta}\right)^{\beta}\right\}$$

Table 6. MLE's for the Three–parameter Generalized Extreme–value Distribution

data case	estimates			
	$\hat{k}$	$\hat{\sigma}$	$\hat{\mu}$	$\log L_{\max}$
1	−.159337	.273013	3.36379	−7.49358
2	.469750	.372719	3.28108	−6.66366
3	.290816	.419075	3.27990	−11.1484
4	.532670	.444848	3.34216	−9.45101
5	.521458	.484733	3.21892	−11.3767
6	.803557	.138803	.433881	17.0683
7	1	5934.10	7298.10	−193.769
8	.867838	836.952	1138.68	−156.264
9	.0843467	.272864	1.64163	−14.2853
10	.0469217	.221493	1.24858	−2.08236
11	.796457	5365.71	8978.02	−194.404
12	.659836	7.75492	22.7406	−71.7569
13	1	67.7800	220.480	−52.1627
14	.454664	46.2825	104.274	−206.258
15	.429187	35.8408	114.308	−196.518
16	1	1.92500	5.02500	−6.61970
17	1	3843.65	3952.65	−185.084
18	.884625	23.2809	153.303	−84.4232
19	.401499	1.94056	5.69794	−603.005
20	.893872	3.29941	5.17955	−108.198
21	.608799	192.343	459.235	−564.101
22	.627235	40.0678	78.7560	−112.850
23	.856607	1419.01	1847.88	−166.998
24	1	4.42110	4.42219	−49.7278
25	1	13.2310	21.9810	−35.8256
26	.461850	11.5823	39.3802	−75.3666
27	.599427	154.719	440.760	−451.383

$$F(x;\sigma,k,\mu) = 1 - \exp\left(-\left\{1 + k\left(\frac{x-\mu}{\sigma}\right)\right\}^{1/k}\right), \quad \text{if} \quad 1 + k(x-\mu)/\sigma > 0$$

Table 7. MLE's for the Three–parameter Gamma Distribution

data case	estimates			
	$\hat{\beta}$	$\hat{\gamma}$	$\hat{\alpha}$	$\log L_{\max}$
1	3.74125	1.94135	.299407	
2	2.17154	7.95813	.127977	
3	4.33001	7.22803	.166022	
4	2.15845	6.84872	.159380	
5	1.98494	5.86468	.192690	
6	.259558	1.38369	.118211	
7	1364.00	1	5934.10	
8	184.000	1	912.550	
9	2.63775	12.9310	.0874588	
10	1.77621	5.78456	.111714	
11	2398.00	1	6170.00	
12	9.37660	2.73365	4.49140	
13	152.700	1	67.7800	
14	−55.3102	11.4094	12.9704	
15	−23.6001	14.2439	9.01542	
16	3.10000	1	1.92500	
17	109.000	1	3843.65	
18	127.211	1	25.0909	
19	−.806239	14.4739	.414525	
20	1.46983	1.27530	2.79098	
21	109.664	3.17909	99.5630	
22	9.23859	2.86051	22.0190	
23	216.000	1	1556.80	
24	.00108736	1	4.42110	
25	8.75000	1	13.2310	
26	−.439507	11.3929	3.24453	
27	3.23805	79.1781	156.979	

$$f(x;\alpha,\beta,\gamma) = \frac{1}{\gamma\Gamma(\beta)}\left(\frac{x-\alpha}{\gamma}\right)^{\beta-1}\exp\left\{-\left(\frac{x-\alpha}{\gamma}\right)\right\},$$
$$(x \geq \alpha, \quad \beta > 0, \quad \gamma > 0),$$

Table 8. MLE's for the Three–parameter Extended Gamma Distribution

data case	estimates			
	$\hat{\lambda}$	$\hat{\sigma}$	$\hat{\mu}$	$\log L_{\max}$
1	−.717709	.417170	3.16000	−7.00027
2	.354482	.361026	3.19000	−7.13821
3	−.371955	.446350	3.13100	−11.2910
4	.382116	.417099	3.25000	−9.88057
5	.412931	.466639	3.11500	−11.9486
6	.850122	.139052	.423125	16.8010
7	1	5934.10	7298.10	−193.769
8	1	912.550	1096.55	−156.325
9	−.278089	.314499	1.50683	−14.8607
10	−.415781	.268683	1.13000	−2.04942
11	1	6170.00	8568.00	−194.549
12	.604823	7.42598	21.6545	−72.3901
13	1	67.7800	220.480	−52.1627
14	.296052	43.8113	92.6750	−206.759
15	.264963	34.0252	104.815	−196.889
16	1	1.92500	5.02500	−6.61970
17	1	3843.65	3952.65	−185.084
18	1	25.0909	152.302	−84.4501
19	.262850	1.57704	5.19355	−573.828
20	.885510	3.15183	5.02917	−108.117
21	.560852	177.521	426.184	−564.167
22	.591259	37.2409	72.2243	−112.914
23	1	1556.80	1772.80	−167.008
24	1	4.42110	4.42219	−49.7278
25	1	13.2310	21.9810	−35.8256
26	.296267	10.9514	36.5250	−75.6499
27	.555723	142.478	413.362	−451.223

$$f(x;\sigma,\mu,\lambda) = \frac{1}{\sigma|\lambda|\Gamma(\lambda^{-2})}\left[\lambda^{-2}\left\{1+\lambda\left(\frac{x-\mu}{\sigma}\right)\right\}\right]^{\lambda^{-2}-1}\exp\left[-\lambda^{-2}\left\{1+\lambda\left(\frac{x-\mu}{\sigma}\right)\right\}\right],$$
$$(\sigma>0, \quad \lambda\neq 0),$$

Table 9. MLE's for the Three−parameter Log−normal Distribution

data case	estimates			
	$\hat{\gamma}$	$\hat{\tau}$	$\hat{\alpha}$	$\log L_{\max}$
1	−.629487	−.552898	3.88959	
2	2.21186	.157035	.950855	
3	−2.70065	−.158285	5.86452	
4	1.96060	.203105	1.24877	
5	3.06990	.143963	.0132649	
6	.209979	.507310	.184977	
7	4032.92	1.02985	950.596	
8	910.098	.702905	−40.3685	
9	−1.41150	−.215901	2.95192	
10	−.857457	−.295268	2.02556	
11	18649.5	.229636	−10573.8	
12	25.6106	.256339	−4.80193	
13	51.3456	.90954	144.115	
14	239.039	.178266	−150.180	
15	203.079	.163841	−101.002	
16	−6.49493	−.207419	11.6597	
17	−9712.67	.312525	57274.7	
18	29.1709	.604313	117.721	
19	7.48023	.203163	−2.44732	
20	3.02169	.763900	1.01621	
21	397.231	.400045	−3.94496	
22	75.2765	.437708	−10.5171	
23	2264.78	.473674	−753.307	
24	−1177690.	−31.283	1178180.	
25	15.5211	.606150	3.54184	
26	60.9843	.17467	−25.3935	
27	319.877	.399337	67.0553	

$$f(x;\alpha,\tau,\gamma) = \frac{1}{\sqrt{2\pi}(x-\alpha)\tau}\exp\Big[-\frac{\{\log(\frac{x-\alpha}{\gamma})\}^2}{2\tau^2}\Big], \quad x>\alpha, \quad \tau>0, \quad \gamma>0$$

Table 10. MLE's for the Three–parameter Extended Log–normal Distribution

data case	estimates			
	$\hat{\lambda}$	$\hat{\sigma}$	$\hat{\mu}$	$\log L_{\max}$
1	−.552898	.348043	3.26010	−7.27016
2	.157035	.347339	3.16271	−7.22968
3	−.158285	.427473	3.16387	−11.3815
4	.203105	.398207	3.20937	−9.96312
5	.143963	.441953	3.08317	−12.0477
6	.507310	.106524	.394956	16.4088
7	1.02985	4153.30	4983.51	−195.012
8	.702905	639.713	869.729	−157.599
9	−.215901	.304745	1.54042	−14.5314
10	−.295268	.253179	1.16810	−2.08291
11	.229636	4282.58	8075.66	−195.625
12	.256339	6.56499	20.8086	−72.6152
13	.909540	46.7009	195.461	−52.6270
14	.178266	42.6126	88.8599	−206.844
15	.163841	33.2727	102.078	−196.947
16	−.207419	1.34717	5.16474	−6.86779
17				
18	.604313	17.6283	146.892	−85.7689
19	.203163	1.51971	5.03290	−569.611
20	.763900	2.30827	4.03790	−108.261
21	.400045	158.910	393.286	−564.393
22	.437708	32.9492	64.7594	−113.020
23	.473674	1072.77	1511.47	−167.939
24				
25	.606150	9.40808	19.0629	−36.6051
26	.174670	10.6521	35.5908	−75.6940
27	.399337	127.739	386.932	−451.363

$$f(x;\sigma,\mu,\lambda) = \frac{1}{\sqrt{2\pi}\sigma\{1+\lambda(\frac{x-\mu}{\sigma})\}} \exp\Big(-\frac{[\log\{1+\lambda(\frac{x-\mu}{\sigma})\}]^2}{2\lambda^2}\Big), \quad \sigma>0, \quad \lambda\neq 0$$

Table 11. MLE's for the Three−parameter Generalized Gamma Distribution

data case	estimates			
	$\hat{b}$	$\hat{a}$	$\hat{p}$	$\log L_{\max}$
1	.185512	3.63704	38.1571	−6.92556
2				
3	1.30368	3.17142	7.20129	−11.3406
4				
5				
6				
7				
8				
9	.619683	1.76924	7.76247	−14.5876
10	.362178	1.43524	10.3597	−2.06712
11	.410969	13832.7	3.9027	−194.748
12	10.1456	2.16084	1.00524	−72.5394
13				
14	.788017	117.794	2.64225	−206.230
15	3.31959	54.244	1.72617	−196.800
16				
17				
18				
19	17.5912	.125108	.771804	−574.482
20				
21				
22	10.5460	1.38988	.603883	−112.969
23	.882967	2168.19	1.63424	−167.332
24	3.96104	.00430215	.247341	−35.0895
26	5.22163	12.2529	1.4849	−75.6266
27				

$$\frac{p}{a}\frac{1}{\Gamma(b)}\left(\frac{x}{a}\right)^{bp-1}\exp\left\{-\left(\frac{x}{a}\right)^{p}\right\}, \quad (x \geq 0;\ a,\ b,\ p > 0)$$

Table 12. MLE's for the Four−parameter Generalized Gamma Distribution

data case	estimates				
	$\hat{a}$	$\hat{c}$	$\hat{p}$	$\hat{b}$	$\log L_{\max}$
1	2.17920	1.48163	26.6909	.132123	−6.80599
2					
3					
4					
5					
6					
7					
8					
9	19.9864	−18.5873	54.0947	1.79838	−14.2394
10	1.76806	−.374205	11.2267	.487745	−2.00359
11	6906.43	2232.05	1.35928	.928641	−194.351
12					
13					
14	137.813	13.2032	3.62604	.360414	−205.833
15	101.040	36.9936	3.09105	.541254	−196.390
16					
17					
18					
19					
20	.837193	1.40455	.701489	2.59691	−108.038
21	261.311	137.095	1.45779	1.29700	−564.072
22	79.0486	16.3183	1.88693	.705228	−112.841
23					
24					
25					
26					
27					

$$f(x;a,b,c,p) = \frac{p}{a}\frac{1}{\Gamma(b)}\left(\frac{x-c}{a}\right)^{bp-1}\exp\left\{-\left(\frac{x-c}{a}\right)^{p}\right\}, \quad (x \geq c;\ a,\ b,\ p > 0)$$

Table 13. MLE's for the Four−parameter Extended Generalized Gamma Distribution

data case	estimates				
	$\hat{k}$	$\hat{\lambda}$	$\hat{\sigma}$	$\hat{\mu}$	$\log L_{\max}$
1	0.0374659	2.75113	0.224618	3.66083	−6.80599
2					
3					
4					
5					
6					
7					
8					
9	0.0184861	0.745692	0.275511	1.39910	−14.2394
10	0.0890736	1.43187	0.225501	1.39385	−2.00359
11	0.735681	1.03771	5272.53	9138.48	−194.351
12					
13					
14	0.275783	1.66571	63.3077	151.016	−205.833
15	0.323515	1.35925	44.4313	138.034	−196.390
17					
18					
19					
20	1.42554	0.620543	0.740588	2.24174	−108.038
21	0.685970	0.878073	157.396	398.406	−564.072
22	0.529960	1.19079	49.8853	95.3669	−112.841
23					
24					
25					
26					
27	0.851181	0.675017	74.6077	296.494	−451.213

$$f(x;\lambda,k,\sigma,\mu) = \frac{|\lambda|}{\sigma}\frac{1}{\Gamma(1/\lambda^2)}\Big\{1+\lambda k\big(\frac{x-\mu}{\sigma}\big)\Big\}^{1/(\lambda^2 k)-1}\exp\Big[-\Big\{1+\lambda k\big(\frac{x-\mu}{\sigma}\big)\Big\}^{1/k}\Big],$$
$$(1+\lambda k\big(\frac{x-\mu}{\sigma}\big) \geq 0;\ \lambda \neq 0,\ k \neq 0,\ \sigma$$

Table 14. MLE's for the Three−parameter Extended Weibull Distribution

data case	estimates			
	$\hat{\alpha}$	$\hat{\eta}$	$\hat{\beta}$	$\log L_{\max}$
1	2.90277	3.12734	8.59342	−7.90657
2	.26468	3.56962	12.9061	−7.42666
3	.595943	3.42640	9.38368	−11.2926
4	.0503903	4.06382	13.1072	−10.2972
5	.241555	3.62484	10.0365	−12.1811
6	.0177532	.754730	6.42248	15.7606
7	.201176	14069.8	1.84213	−195.848
8	1.30159	1097.32	1.51182	−157.157
9	16.6435	1.12043	3.20180	−12.0336
10	6.65409	.943048	3.34779	−2.32922
11	1.53581	8568.15	1.92288	−194.92 0
12	.22436	29.8040	4.61834	−72.5814
13	.00125738	537.338	6.85203	−54.5110
14	1.22185	99.2871	2.17935	−206.26
15	.248745	144.648	4.33942	−196.829
16	1.01366	5.52973	4.30149	−6.82430
17				
18				
19				
20				
21				
22				
23				
24				
25				
26				
27	.0118887	912.349	5.22532	−452.248

$$G(x;\alpha,\eta,\beta) = 1 - \frac{\alpha w}{1-(1-\alpha)w}$$

$$w = \exp(-z), \quad z = y^{\beta}, \quad y = x/\eta$$

Table 15. MLE's for the Two–parameter Extended Exponential Distribution

data case	estimates		
	$\hat{\alpha}$	$\hat{\eta}$	$\log L_{\max}$
1	1317140.	.227170	−10.0316
2	2962860.	.213439	−8.28739
3	188868.	.258540	−12.3003
4	983298.	.234547	−10.6657
5	113250.	.267093	−12.9388
6	479.462	.0660682	14.1847
7	2.6636	4634.5	−197.047
8	5.03692	549.812	−157.812
9	8247.22	.170931	−16.0134
10	2121.1201	.150160	−4.43979
11	15.7747	2948.85	−195.641
12	208.116	3.99181	−73.5531
13	345.402	35.2731	−56.2003
14	25.3395	27.5597	−206.957
15	178.924	19.9386	−197.969
16	440.172	.830316	−7.06792
19	752351.	11.1418	−87.9245
20	496.609	.818332	−562.664
21	8.7603	1.95912	−116.593
22	55.4428	100.976	−570.150
23			
24			
25			
26			
27	157.766	78.9034	−456.627

$$G(x;\alpha,\eta) = 1 - \frac{\alpha v}{1-(1-\alpha)v}$$
$$v = \exp(-y), \quad y = x/\eta$$

Table 16. MLE's for the Three−parameter Log-gamma Distribution (log-Pearson type III)

data case	estimates			
	$\hat{\gamma}$	$\hat{\beta}$	$\hat{\alpha}$	$\log L_{\max}$
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				
11				
12				
13				
14				
15				
16				
17				
18				
19				
20				
21				
22				
23				
24				
25				
26				
27	0.0318944	105.499	2.60492	−451.408

$$f(x;\alpha,\beta,\gamma) = \frac{1}{\gamma\Gamma(\beta)x}\left(\frac{\log(x)-\alpha}{\gamma}\right)^{\beta-1}\exp\left\{-\left(\frac{\log(x)-\alpha}{\gamma}\right)\right\},$$
$$(\log(x) \geq \alpha, \quad \beta > 0, \quad \gamma > 0),$$

Log-likelihood values for various data cases

data case	model										
	EXP2	G2P	W2P	GEV	ELN3P	EGM3P	GGD3P	EGGD4P	UNIFORM	EW3P	EE2P
1	−21.1654	−7.74824	−8.18392	−7.49358	−7.27016	−7.00027	−6.92556	−6.80599	−9.40007	−7.90657	−10.0316
2	−9.44735	−8.29778	−7.80851	−6.66366	−7.22968	−7.13821		−3.64643	−3.64643	−7.42666	−8.28739
3	−13.7058	−11.6892	−11.3481	−11.1484	−11.3815	−11.2910	−11.3406	−6.72944	−6.72944	−11.2926	−12.3003
4	−11.3843	−12.7342	−11.5332	−9.45101	−9.96312	−9.88057		−8.41469	−9.40007	−10.2972	−10.6657
5	−13.2905	−13.3312	−12.5830	−11.3767	−12.0477	−11.9486		−8.10930	−8.10930	−12.1811	−12.9388
6	16.8874	9.54874	13.2640	17.0683	16.4088	16.8010		17.7047	14.8888	15.7606	14.1847
7	−193.769	−204.433	−196.375	−193.769	−195.012	−193.769		−193.769	−197.082	−195.848	−197.047
8	−156.325	−161.148	−157.082	−156.264	−157.599	−156.325		−154.992	−154.480	−157.157	−157.812
9	−60.2195	−14.9561	−15.2068	−14.2853	−14.5314	−14.8607	−14.5876	−14.2394	−33.0579	−12.0336	−16.0134
10	−33.3759	−2.17102	−3.34939	−2.08236	−2.08291	−2.04942	−2.06712	−2.00359	−9.89512	−2.32922	−4.43979
11	−194.549	−197.499	−194.916	−194.404	−195.625	−194.549	−194.748	−194.351	−192.857	−194.920	−195.641
12	−71.8834	−75.4281	−73.0153	−71.7569	−72.6152	−72.3901	−72.5394	−69.6488	−68.9809	−72.5814	−73.5531
13	−52.1627	−60.2121	−57.3013	−52.1627	−52.6270	−521.627		−52.0876	−55.9805	−54.5110	−56.2003
14	−214.101	−212.187	−206.270	−206.258	−206.844	−206.759	−206.230	−205.833	−209.881	−206.260	−206.957
15	−206.302	−202.272	−197.323	−196.518	−196.947	−196.889	−196.800	−196.390	−198.848	−196.829	−197.969
16	−6.61970	−6.91296	−6.82431	−6.61970	−6.86779	−6.61970		−5.23333	−5.23333	−6.82430	−7.06792
17	−185.084	−225.292	−175.538	−185.084		−185.084			−217.745		−178.765
18	−84.4501	−91.0974	−89.5248	−84.4232	−85.7689	−84.4501		−83.6658	−86.1364		−87.9245
19	−754.400	−768.029	−613.510	−603.005	−569.611	−573.828	−574.482	−677.543	−896.015		−562.664
20	−108.531	−144.682	−115.860	−108.198	−108.261	−108.117		−108.038	−140.825		−116.593
21	−574.764	−594.926	−570.278	−564.101	−564.393	−56.4167		−564.072	−596.604		−570.150
22	−114.893	−120.038	−113.691	−112.850	−113.020	−112.914	−112.969	−112.841	−116.076		
23	−167.008	−173.887	−167.337	−166.998	−167.939	−167.008	−167.332	−166.118	−169.259		
24	−49.7278	−88.5559	−35.7180	−49.7278		−49.7278	−35.0895		−81.0796		
25	−35.8256	−39.2035	−37.0484	−35.8256	−36.6051	−35.8256		−35.5246	−35.6417		
26	−78.5972	−78.2604	−75.9982	−75.3666	−75.6940	−75.6499	−75.6266	−74.8728	−75.5011		
27		−797.408	−459.135	−451.383	−451.363	−451.223		−451.213		−452.248	−456.627

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