Feedback Coordination Control for a Class of Two-Level Descriptor Systems

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In this paper, we consider optimal coordination problems for a class of two-level descriptor systems consisting of N subsystems. Each subsystems is with different cost functional, and they are interconnected by the strategy of the coordinator. The objective of the coordinator is to optimize N different cost functionals simultaneously by using one coordinating strategy. Sufficient conditions for the existence of the descriptor variable feedback coordinating strategy are derived for both the finite-and the infinite-horizon problems. A simple numerical example is solved by using the method given in this paper.

Keyword：descriptor systems, coordination control, different cost functional,

本論文では，N 個の部分システムで構成される二階層ディスクリプタシステムに対する最適調整問題を考泰する。各部分システムは各々異なる費用関数を持ち，調整者の戦略によって相互に接続されている。調整者の目的は一つの調整戦略によって同時にN 個の異なる費用関数を最適化することである。ディスクリプタ可変フィードバックの存在のための十分条件を有限と非有限限界問題の双方に対して求めている。簡単な数値例が本論分の方法を使って解かれている。

1. Introduction

In this paper, We study the problem of performance optimization of dynamic systems upon which control action is exerted at two different levels. The system considered is composed of finite N linear time-invariant descriptor subsystems[1)−3)] each controlled by a lower-level decision maker and a coordinator. For general cases, each subsystem will have a different cost functional whose realized value depends on the controls of both the coordinator and the other lower-level decision makers. This dependence is in fact due to the interconections that exist

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among the subsystems, and also between these subsystems and the coordinator. The former type of interconnections is mainly via the states of the subsystems. whereas the latter type of interconnections could be via the decision of the coordinator. In this paper, only the latter type of interconnections is considered. Therefore, the realized cost value of each subsystem is dependent on the controls of itself and the coordinator. The coordinator's objective is to optimize the performance of the over all system, and the lower-level decision maker's objective is to optimize the performance of its corresponding subsystem. The purpose of this paper is to find a single coordinating strategy to optimize N subsystems simutaniously even if each lower-level decision maker behaves according to its own interest.

Such coordination problems often arise in practice, such as in economic systems, power networks and industrial plants[9]. As a simple example, we know that a central government is preferred to make one common economic policy to influence different local areas among which there exist no direct connections. Furthermore, the feature that the subsystems are described by the descriptor equations would strengthen the usefulness of this paper since the descriptor systems are initially developped for modeling economic systems and electrical networks.

Because each subsystem is described by descriptor equation, it is possible to find a descriptor variable feedback coordinating strategy to coordinate different subsystems. That is impossible in state space formulation.

The paper is organized as follows. In the next section, we give the problem formulation and derive the team-optimal controls for each subsystem independently. Section 3 is concerned with the derivation of the sufficient conditions for the existence of the descriptor variable feedback coordinating strategy when N = 2 for the reason of notation simplicity. A numerical example is solved to show the validity of the method given in this section. In Section 4, the coordination problem for an infinite-time interval case is studied, a static descriptor variable feedback coordinating strategy is obtained. Section 5 is the conclusion section.

2. Problem formulation

Consider a two-level system consisting of N linear time-invariant descriptor subsystems.

$$S_i : E^i\dot{x}_i = A^i x_i + B^i v_i + C^i u_i, \quad E^i x_i(0_-) = E^i x_{io}, \quad i = 1, \cdots, N, \tag{2.1}$$

where $x_i \varepsilon R^n$ is the descriptor vector of S_i. $v_i \varepsilon R^m$ is the decision vector controlled by the lower decision maker of S_i (called decision maker i). $u_i \varepsilon R^1$ ($i = 1, \cdots, N$) are the decision vectors controlled by the coordinator. E^i may be a singular matrix with rank$(E^i) = r^i \leqq n$. The pencil $\lambda E^i - A^i$ is assumed to be regular, i.e., $\det(\lambda E^i - A^i) \neq 0$. $E^i x_{io}$ is a specified initial condition for S_i. The cost functional for S_i is defined by

$$J_i = \frac{1}{2} x_i^T(t_f) E^{iT} Q_f^i E^i x_i(t_f) + \frac{1}{2}\int_{t_0}^{t_f} \{ x_i^T Q^i x_i + v_i^T R_1^i v_i + u_i^T R_2^i u_i \} dt, \quad i = 1, \cdots, N, \tag{2.2}$$

where, the weighting matrices Q_f^i and Q^i are assumed to be symmetric positive semidefinite matrices. R_1^i and R_2^i are assumed to be symmetric positive definite matrices. The time interval $[t_o, t_f]$ is fixed. The superscript ^{T} denotes the transpose of the matrix. The superscrip i is used to differ N different subsystems.

Assumption 2. 1.

The subsystems given in (2. 1) are with similar system structures, that is, $r^1 = r^2 = \cdots = r^N = r$.

The initial condition on the part of x_i (t) in the orthogonal complement of the kernel of E^i is known by both the coordinator and the decision maker i. We also assume that the decision maker i has access to perfect descriptor variable information on S_i, and the coordinator has access to perfect descriptor variable information on all subsystems. As a two-level decision making problem, the coordinator on the upper level determines a strategy and applies it to the subsystems. Knowing what is the coordinator's strategy, each decision makers on the lower level chooses his own strategy to optimize his own cost functional. The strategy of the coordinator is denoted by $\gamma_c(\eta)\ \varepsilon\ \Gamma$, whose realization is u_i (i = 1, ⋯, N) depending on what information η is received by the coordinator. The strategy of the decision makeri is denoted by $\gamma_i \varepsilon \Gamma_i$, whose realization is v_i. In this paper, we assume that the strategy spaces Γ and Γ_i are all composed of the descriptor variable feedback strategies.

Now, let $(v_i^*, u_i^*)\ \varepsilon\ V_i \times U_i$ be a pair of decisions which globally minimizes $J_i(v_i, u_i)$, where V_i and U_i represent decision spaces of corresponding decision makers.

Definition 2. 1.

A strategy $\gamma_c^*(\eta)\ \varepsilon\ \Gamma$ is said to be a coordinating strategy of the coordinator if the following equations are satisfied,

$$\arg\min_{v_i} J_i(\gamma_c^*, v_i) = v_i^*, \qquad i = 1, \cdots, N, \tag{2. 3}$$

$$\gamma_c^* = u_i^*, \text{ when } \eta \text{ is the information of } S_i, \qquad i = 1, \cdots, N. \tag{2. 4}$$

Remark 2. 1.

In Definition 2. 1, that a strategy equals a decision means that the realization of that strategy equals the corresponding decision.

Now, the descriptor variable feedback form of $(v_i^*, u_i^*)\ \varepsilon\ V_i \times U_i$ can be obtained by solving N independent team-optimal control problems respectively. According to Theorem 1. 2[8], there exist two nonsingular matrices H^i, T^i for the ith (i = 1,⋯, N) subsystem such that

$$H^iE^iT^i = \begin{bmatrix} Ir & 0 \\ 0 & 0 \end{bmatrix}, \quad r = \text{rank } E^i, \tag{2. 5}$$

In the following, we call the transformation (2. 5) as the ith reduced-order transformation to simplify our expression. Then, the system (2. 1) is transformed to the following form by coordinate transformation $x_i = T^i \begin{bmatrix} X_{i1} \\ X_{i2} \end{bmatrix}$,

$$\begin{bmatrix} Ir & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{X}_{i1} \\ \dot{X}_{i2} \end{bmatrix} = \begin{bmatrix} A^i_{11} & A^i_{12} \\ A^i_{21} & A^i_{22} \end{bmatrix} \begin{bmatrix} X_{i1} \\ X_{i2} \end{bmatrix} + \begin{bmatrix} B^i_1 \\ B^i_2 \end{bmatrix} v_i + \begin{bmatrix} C^i_1 \\ C^i_2 \end{bmatrix} u_i, \tag{2. 6}$$

where

$$H^iA^iT^i = \begin{bmatrix} A^i_{11} & A^i_{12} \\ A^i_{21} & A^i_{22} \end{bmatrix}, \quad H^iB^i = \begin{bmatrix} B^i_1 \\ B^i_2 \end{bmatrix}, \quad H^iC^i = \begin{bmatrix} C^i_1 \\ C^i_2 \end{bmatrix}, \tag{2. 7}$$

and

$$x_{i1}(0)=\begin{bmatrix} Ir & 0 \\ 0 & 0 \end{bmatrix}\left(T^{i}\right)^{-1} x_{i0}=x_{i10}, \tag{2.8}$$

The cost functional (2.2) is changed to

$$J_i=\frac{1}{2}x_{i1}^{T}(t_f)\,Q_{11f}^{i}\,X_{i1}(t_f)+\frac{1}{2}\int_{t_0}^{t_f}\left\{\left[x_{i1}^{T}\ \ x_{i2}^{T}\right]\begin{bmatrix} Q_{11}^{i} & Q_{12}^{i} \\ Q_{12}^{i\,T} & Q_{22}^{i} \end{bmatrix}\begin{bmatrix} X_{i1} \\ X_{i2} \end{bmatrix}+V_i^{T}R_1^{i}v_i++u_i^{T}R_2^{i}u_i\right\}dt, \tag{2.9}$$

where

$$T^{iT}Q^{i}T^{i}=\begin{bmatrix} Q_{11}^{i} & Q_{12}^{i} \\ Q_{12}^{i\,T} & Q_{22}^{i} \end{bmatrix}=\begin{bmatrix} C_{11}^{i\,T} \\ C_{12}^{i\,T} \end{bmatrix}\left[C_{11}^{i}\ \ C_{12}^{i}\right],$$

$$(H^{i})^{-T}Q_{f}^{i}(H^{i})^{-1}=\begin{bmatrix} Q_{11f}^{i} & Q_{12f}^{i} \\ Q_{12f}^{i\,T} & Q_{22f}^{i} \end{bmatrix}, \tag{2.10}$$

Assumption 2.2.

$[A_{22}^{i}\ B_2^{i}\ C_2^{i}]$ and $[A_{22}^{i\,T}\ C_{12}^{i\,T}]$, are of full row rank.

If Assumption 2.2 is satisfied, then the impulsive modes of the ith subsystem are controllable and observable. Under Assumption 2.2, we compute (v_i^*, u_i^*) in the following way.

$$\begin{bmatrix} v_i^* \\ u_i^* \end{bmatrix}=-\begin{bmatrix} R_1^{i-1} & 0 \\ 0 & R_2^{i-1} \end{bmatrix}\begin{bmatrix} B_1^{iT} & B_2^{i\,T} \\ C_1^{iT} & C_2^{i\,T} \end{bmatrix}G^{i}(t)(T^{i})^{-1}x_i, \tag{2.11}$$

where

$$G^{i}(t)=\begin{bmatrix} P^{i}(t) & 0 \\ L_2^{i}(t)-F^{i}L_1^{i}(t) & F^{i} \end{bmatrix}, \tag{2.12}$$

$$\begin{bmatrix} L_1^{i}(t) \\ L_2^{i}(t) \end{bmatrix}=-M_4^{i-1}M_3^{i}\begin{bmatrix} Ir \\ P^{i}(t) \end{bmatrix}, \tag{2.13}$$

and $P^{i}(t)$ satisfies the Riccati differential equation.

$$-\dot{P}^{i}(t)=P^{i}(t)A_r^{i}+A_r^{iT}P^{i}(t)-P^{i}(t)W_r^{i}P^{i}(t)+Q_r^{i},\quad P^{i}(t_f)=Q_{11f}^{i}, \tag{2.14}$$

respectively. F^{i} is an arbitrary constant matrix which makes $A_{22}^{i}-W_3^{i}F^{i}$ invertible. The other related matrices above are defined as

$$\begin{bmatrix} A_r^{i} & -W_r^{i} \\ -Q_r^{i} & -A_r^{i} \end{bmatrix}=M_1^{i}-M_2^{i}M_4^{i-1}M_3^{i}.$$

$$M_1^{i}=\begin{bmatrix} A_{11}^{i} & -W_1^{i} \\ -Q_{11}^{i} & -A_{11}^{i\,T} \end{bmatrix},\quad M_2^{i}=\begin{bmatrix} A_{12}^{i} & -W_2^{i} \\ -Q_{12}^{i} & -A_{21}^{i\,T} \end{bmatrix},$$

$$M_3^{i}=\begin{bmatrix} A_{13}^{i} & -W_2^{iT} \\ -Q_{12}^{i\,T} & -A_{12}^{i\,T} \end{bmatrix},\quad M_4^{i}=\begin{bmatrix} A_{22}^{i} & -W_3^{i} \\ -Q_{13}^{i} & -A_{22}^{i\,T} \end{bmatrix},$$

$$W_1^{i}=B_1^{i}R_1^{i\,-1}B_1^{i\,T}+C_1^{i}R_2^{i\,-1}C_1^{i\,T},$$

$$W_2^{i}=B_1^{i}R_1^{i\,-1}B_2^{i\,T}+C_1^{i}R_2^{i\,-1}C_2^{i\,T},$$

$$W_3^{i}=B_2^{i}R_1^{i\,-1}B_2^{i\,T}+C_2^{i}R_2^{i\,-1}C_2^{i\,T}, \tag{2.15}$$

Lemma 2.1.

The lower triangular block matrix $G^i(t)$ will be the team-optimal feedback gain of S_i if F^i makes $A^i_{22} - W^i_3 F^i$ invertible.

Proof.

It is the corollary of Lemma 5[11].

Lemma 2. 2.

The team-optimal trajectories x_{i1}^* and x_{i2}^* satisfy the equation

$$x_{i2}^* = L^i_1(t) X_{i1}^*, \qquad i = 1, \cdots, N. \tag{2. 16}$$

Proof.

The proof follows from Ref. 11.

3. Sufficient Conditions for the Existence of Descriptor Variable Feedback Strategy

In order to simiplify the notation and give an explicit expression of the descriptor variable feedback coordinating strategy for a special case, we derive the sufficient conditions by specifying N=2 in this and the next sections without any loss of generality. As mentioned in the introduction, only the interconnections between the coordinator and each subsystem are considered, therefore, the coordinator may use different strategies to coordinate two subsystems respectively. Then the problem becomes two independent incentive problems. In such a case, the coordinating strategies can be designed as follows.

Assumption 3. 1.

$[A^i_{22}\ C^i_2]$ and $[A^{i\,T}_{22}\ C^{i\,T}_{12}]$, $i = 1, 2$, are of full row rank.

Lemma 3. 1.

Under the Assumption 3. 1. there exists time-varing matrices $F^i(t)$, $i = 1, 2$, that

$$\gamma_i^* = u_i^* = -R^{i\,-1}_2 [C^{i\,T}_1\ C^{i\,T}_2] K^i_1(t) (T^i)^{-1} x_i, \qquad i = 1, 2, \tag{3. 1a}$$

$$V_i^* = -R^{i\,-1}_1 [B^{i\,T}_1\ B^{i\,T}_2] K^i_2(t) (T^i)^{-1} x_i, \qquad i = 1, 2, \tag{3. 1b}$$

constitute the team-optimal controls for two subsystems respectivelty, where

$$K^i_1(t) = \begin{bmatrix} P^i(t) & 0 \\ L^i_2(t)\text{-}F^i(t)L^i_1(t) & F^i(t) \end{bmatrix}, \quad i = 1, 2, \tag{3. 2a}$$

and

$$K^i_2(t) = \begin{bmatrix} P^i(t) & 0 \\ L^i_2(t) & 0 \end{bmatrix}, \qquad i = 1, 2. \tag{3. 2b}$$

Proof.

See Appendix for the proof.

According to Lemma 3. 1, the coordinator can induce the behavior of two lover decision makers by adopting two different team-optimal strateg-ies γ_1^* and γ_2^* in (3. 1a) respectively.

But each strategy is only useful to corresponding subsystem. In the following, we want to find a single strategy to coordinate two subsystems.

Theorem 3.1.

If there exist two time-varing matrices $F^{1*}(t)$ and $F^{2*}(t)$ such that

(a). $A^1_{22}-W^1_{32}F^{1*}(t)$ and $A^2_{22}-W^2_{32}F^{2*}(t)$ are invertible for $t\varepsilon\ [t_0, t_f]$,

(b). $R_2^{1\,-1}\{C_1^{1\,T}P^1(t)+C_2^{1\,T}[L_2^1(t)-F^{1*}(t)L_1^1(t)]\}T^1_{11}+R_2^{1\,-1}C_2^{1\,T}F^{1*}(t)T^1_{21}$

$$=R_2^{2\,-1}\{C_1^{2\,T}P^2(t)+C_2^{2\,T}[L_2^2(t)-F^{2*}(t)L_1^2(t)]\}T^2_{11}+R_2^{2\,-1}C_2^{2\,T}F^{2*}(t)T^2_{21}, \tag{3.3a}$$

$R_2^{1\,-1}\{C_1^{1\,T}P^1(t)+C_2^{1\,T}[L_2^1(t)-F^{1*}(t)L_1^1(t)]\}T^1_{12}+R_2^{1\,-1}C_2^{1\,T}F^{1*}(t)T^1_{22}$

$$=R_2^{2\,-1}\{C_1^{2\,T}P^2(t)+C_2^{2\,T}[L_2^2(t)-F^{2*}(t)L_1^2(t)]\}T^2_{12}+R_2^{2\,-1}C_2^{2\,T}F^{2*}(t)T^2_{22}, \tag{3.3b}$$

where T^i_{11}, T^i_{12}, T^i_{21} and T^i_{22} $(i=1, 2)$ are the block matrices with dimensions $r\times r$, $r\times(n-r)$, $(n-r)\times r$ and $(n-r)\times(n-r)$, respectively, defined by

$$(T^i)^{-1}=\begin{bmatrix} T^i_{11} & T^i_{12} \\ T^i_{21} & T^i_{22} \end{bmatrix}, \qquad i=1, 2, \tag{3.4}$$

then there exists the descriptor variable feedback coordinating strategy γ_c^* to coordinate two different subsystems.

Proof.

From Lemma 3.1, we know that

$$\gamma_c^{i*}=-R_2^{i\,-1}[C_1^{i\,T}\ C_2^{i\,T}]K_1^{i*}(t)(T^i)^{-1}x_i, \qquad i=1, 2, \tag{3.5}$$

constitutes the team-optimal control of the coordinator for S_i, where $K_1^{i*}(t)$ is defined by (3.2a) and $F^i(t)$ is substituted by $F^{i*}(t)$.

Because that the condition (b) is satisfied, it is easy to prove that the feedback matrix of γ_c^{1*} is equal to the feedback matrix of γ_c^{2*}. Therefore, either of γ_c^{1*} and γ_c^{2*} can be used as the coordinating strategy γ_c^*.

Theorem 3.1 gives the sufficient conditions for the existence of the coordinating strategy γ_c^*. It is complex to get the explicity solutions of $F^{1*}(t)$ and $F^{2*}(t)$. But, for a special case when $E^1=E^1$, we can obtain the explicit expressions of $F^{1*}(t)$ and $F^{2*}(t)$ easily. In this case, the condition (b) becomes

$R_2^{1\,-1}\{C_1^{1\,T}P^1(t)+C_2^{1\,T}[L_2^1(t)-F^{1*}(t)L_1^1(t)]\}$

$$=R_2^{2\,-1}\{C_1^{2\,T}P^2(t)+C_2^{2\,T}[L_2^2(t)-F^{2*}(t)L_1^2(t)]\}, \tag{3.6a}$$

$$R_2^{1\,-1}C_2^{1\,T}F^{1*}(t)=R_2^{2\,-1}C_2^{2\,T}F^{2*}(t). \tag{3.6b}$$

Using (3.6), we get

$$F^{1*}(t)=(C_2^1C_2^{1\,T})\ +C_2^1R_2^1R_2^{2\,-1}C_2^{2\,T}F^{2*}\ (t)\ , \tag{3.7a}$$

$$F^{2*}(t)=(C_2^2C_2^{2\,T})\ +C_2^2R_2^2(H^2(t)-H^1(t))L(t)^T[L(t)L(t)^T]^+, \tag{3.7b}$$

where

$$H^1(t)=R_2^{1\,-1}[C_1^{1\,T}P^1(t)+C_2^{1\,T}L_2^1(t)], \tag{3.8a}$$

$$H^2(t)=R_2^{2\,-1}[C_1^{2\,T}P^2(t)+C_2^{2\,T}L_2^2(t)], \tag{3.8b}$$

$$L(t)=[L_1^2(t)-L_1^1(t)], \tag{3.8c}$$

and the superscript + denotes the Moore-Penrose generalized inverse. Obviously, $F^{1*}(t)$ and

$F^{2*}(t)$ may not be uniquely determined. For the solvable problem, we need $2(n-r)(n-r) \geqq n \times l$ to make (3. 3) be solvable.

To the end of this section, we solve a simple numerical example.

Example.

Given two subsystems described by

$$S_1: \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{x}_{11} \\ \dot{x}_{12} \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix} \begin{bmatrix} x_{11} \\ x_{12} \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \end{bmatrix} v_1 + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u_1, \quad x_{11}(0_-) = x_{110},\ x_{12}(0_-) = x_{120}, \tag{3. 9a}$$

$$S_2: \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{x}_{21} \\ \dot{x}_{22} \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 3 & 0 \end{bmatrix} \begin{bmatrix} x_{21} \\ x_{22} \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} v_2 + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_2, \quad x_{21}(0_-) = x_{210},\ x_{22}(0_-) = x_{220}, \tag{3. 9b}$$

and the cost functionals for two subsystems are

$$J_1 = x_{11}^2(1) + \frac{1}{2}\int_0^1 \{y_1^T y_1 + 2v_1^2 + u_1^2\} dt, \tag{3. 10a}$$

$$J_2 = (3/2)x_{21}^2(1)\frac{1}{2}\int_0^1 \{y_2^T y_2 + v_2^2 + u_2^2\} dt, \tag{3. 10b}$$

where $t_0 = 0$, $t_f = 1$ and

$$y_1 = [1 \quad 2]\begin{bmatrix} x_{11} \\ x_{12} \end{bmatrix}, \quad y_2 = [3 \quad 1]\begin{bmatrix} x_{21} \\ x_{22} \end{bmatrix}. \tag{3. 11}$$

From (2. 15),

$$M_1^1 = \begin{bmatrix} 1 & -1.5 \\ -1 & -1 \end{bmatrix}, \quad M_2^1 = \begin{bmatrix} 2 & -2 \\ -2 & 3 \end{bmatrix}, \quad M_3^1 \begin{bmatrix} 3 & -2 \\ -2 & -2 \end{bmatrix},$$

$$M_4^1 = \begin{bmatrix} 0 & -3 \\ -4 & 0 \end{bmatrix}, \quad M_1^2 = \begin{bmatrix} 2 & -1 \\ -9 & -2 \end{bmatrix}, \quad M_2^2 = \begin{bmatrix} 3 & -1 \\ -3 & -3 \end{bmatrix},$$

$$M_3^2 = \begin{bmatrix} 3 & -1 \\ -3 & 3 \end{bmatrix}, \quad M_4^2 = \begin{bmatrix} 0 & -2 \\ -1 & 0 \end{bmatrix}. \tag{3. 12}$$

$p^1(t)$ and $p^2(t)$ in (2. 14) (in this example, small letters are used to denote scalar functions concerned) satisfy the following two differential equations respectively,

$\dot{p}^1(1) = 1.1667[p^1(t)]^2 + 4p^1(t) - 3, \ p^1(1) = p_{1f}$,

$\dot{p}^2(t) = 9.5[p^2(t)]^2 + 17p^2(t) - 4.5, \ p^2(1) = p_{2f}$.

They are shown in Fig. 1 when $p_{1f} = 2$ and $p_{2f} = 3$.

Using (2. 13), we can compute $l_1^i(t)$ and $l_2^i(t)$ for $i = 1, 2$, which are shown in Fig. 2 and Fig. 3 respectively. $f^{1*}(t)$ and $f^{2*}(t)$ are computed from (3. 7), and are shown in Fig. 4. The coordinating strategy is

$$\gamma_c^*(\eta) = [g^*(t) f^*(t)]\,\eta, \tag{3. 11}$$

where $g^*(t)$ is shown in Fig. 1, $f^*(t)$ is shown in Fig. 4 and $\eta = [x_{11} \ x_{12}]^T$ or $\eta = [x_{21} \ x_{22}]^T$. In Fig. 5 and Fig. 6, we show the optimal strategies of the coordinator and the lower decision makers of two subsystems respectively, in the form of the realizations of the strategies. In Fig. 7 and Fig. 8, the cooresponding optimal trajectories are shown respectively for two subsystems, where we can see that x_{12}^* and x_{22}^* may undergo step change when t moves from $t = 0_-$ to $t = 0_+$.

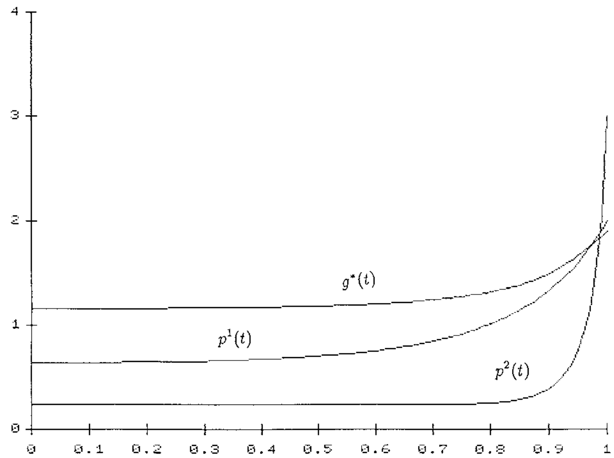

Fig. 1 $g^*(t)$, $p^1(t)$ and $p^2(t)$ for constructing the coordinating strategy

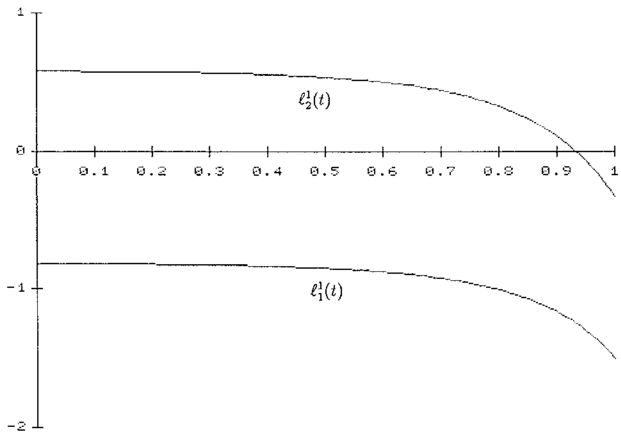

Fig. 2 $l_1^1(t)$ and $l_2^1(t)$ for constructing the coordinating strategy

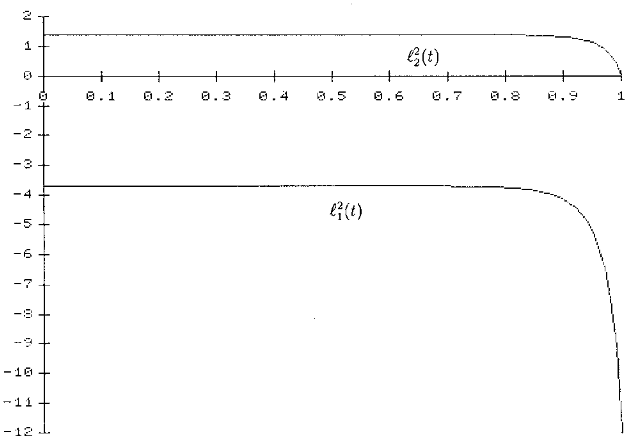

Fig. 3 $l_1^2(t)$ and $l_2^2(t)$ for constructing the coordinating strategy

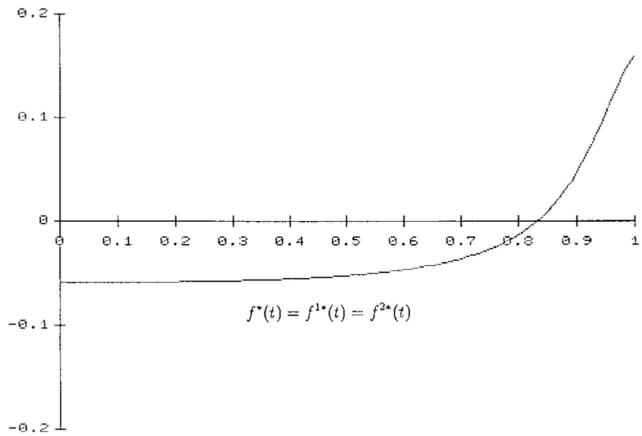

Fig. 4 f*(t) in the coordinating strategy

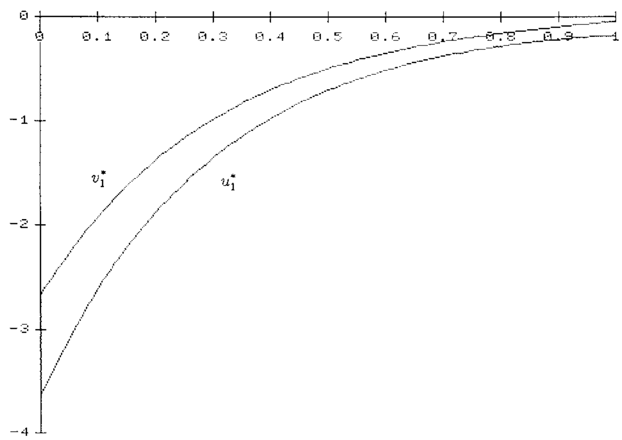

Fig. 5 The optimal strategies of the coordinator and the decision maker 1 when $x_{110}=3$

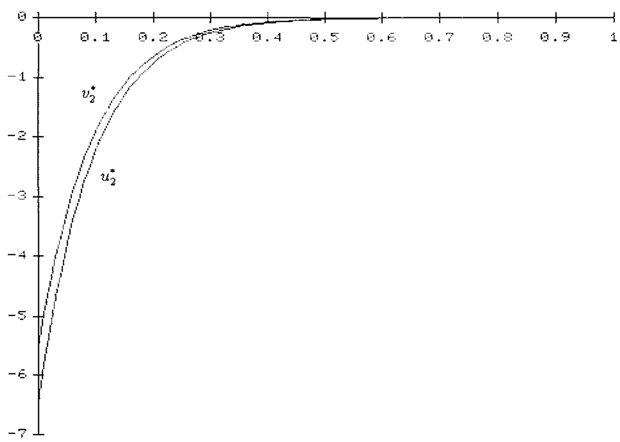

Fig. 6 The optimal strategies of the coordinator and the decision maker 2 when $x_{210}=4$

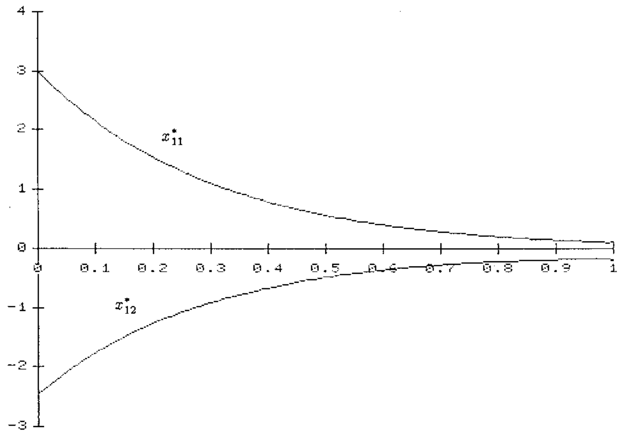

Fig. 7 The optimal descriptor trajectories for S_1 when $x_{110}=3$ and $x_{120}=2$

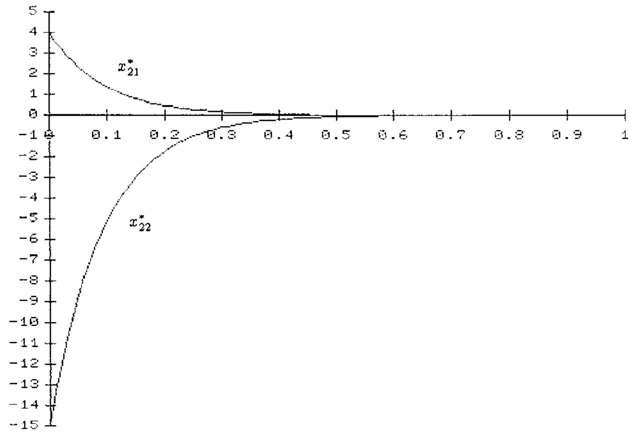

Fig. 8 The optimal descriptor trajectories for S_2 when $x_{210}=4$ and $x_{220}=-1$

4. The infinite-horizon problem

In this section, we derive the sufficient conditions under which the coordinator can coordinate two subsystems by one coordinating strategy for the case when $t_f \to \infty$. For a meaningful formulation, let $Q^i_f=0$ in (2. 2) for i = 1, 2.

Assume that the subsystems (2. 1) and (2. 2) are stabilizable and detectable for finite modes and controllable and observable for impulsive modes when $B^i=0$, i = 1, 2. In the former case, we require that

$$\mathrm{rank}[sE-A^iC^i]=n, \quad \mathrm{rank}[sE-A^i/\sqrt{Q^i}\,]=n, \qquad i=1,2, \tag{4. 1}$$

for any complex s with non-negative real part. The Assumption 3. 1 is sufficient for the latter case. Since the infinite-horizon problem can be considered as the limiting case of the finite-horizon problem, in the following we only state the corresponding conclusions without proof.

Lemma 4. 1.

If the assumptions stated above are satisfied, there exist constant matrix F^i, i = 1, 2, such that

$$\gamma_i^* = u_i^* = -R_2^{i\,-1}[C_1^{i\,T}\ C_2^{i\,T}]K_1^i(T^i)^{-1}x_i,\quad i=1,2, \tag{4. 2a}$$

$$v_i^* = -R_1^{i\,-1}[B_1^{i\,T}\ B_2^{i\,T}]K_2^i(T^i)^{-1}x_i,\quad i=1,2, \tag{4. 2b}$$

constitute the team-optimal controls for tow subsystems respectively, where,

$$K_1^i = \begin{bmatrix} P^i & 0 \\ L_2^i - F^i L_1^i & F^i \end{bmatrix},\quad K_2^i = \begin{bmatrix} P^i & 0 \\ L_2^i & 0 \end{bmatrix},\quad i=1,2, \tag{4. 3}$$

and P^i satisfies the Riccati algebraic equations

$$P^i A_r^i + A_r^{i\,T} P^i - P^i W_f^i P^i + Q_r^i = 0 \tag{4. 4}$$

for i = 1, 2. F^i is an arbitrary constant matrix making $A_{22}^i - W_{32}^i F^i$ invertible for i = 1, 2.

$$\begin{bmatrix} L_1^i \\ L_2^i \end{bmatrix} = -M_4^{i\,-1} M_3^i \begin{bmatrix} I_r \\ P^i \end{bmatrix},\quad i=1,2. \tag{4. 5}$$

Theorem 4. 1.

If there exist two constant matrices F^{1*} and F^{2*} such that

(a). $A_{22}^1 - W_{32}^1 F^{1*}$ and $A_{22}^2 - W_{32}^2 F^{2*}$ are inverti-ble,

(b). $$R_2^{1\,-1}[C_1^{1\,T}P^1 + C_2^{1\,T}(L_2^1 - F^{1*}L_1^1)]T_{11}^1 + R_2^{1\,-1}C_2^{1\,T}F^{1*}T_{21}^1$$
$$= R_2^{2\,-1}[C_1^{2\,T}P^2 + C_2^{2\,T}(L_2^2 - F^{2*}L_1^2)]T_{11}^2 + R_2^2 - C_2^{2\,T}F^{2*}T_{21}^2. \tag{4. 6a}$$

$$R_2^{1\,-1}[C_1^{1\,T}P^1 + C_2^{1\,T}(L_2^1 - F^{1*}L_1^1)]T_{12}^1 + R_2^{1\,-1}C_2^{1\,T}F^{1*}T_{22}^1$$
$$= R_2^{2\,-1}[C_1^{2\,T}P^2 + C_2^{2\,T}(L_2^2 - F^{2*}L_1^2)]T_{12}^2 + R_2^{2\,-1}C_2^{2\,T}F^{2*}T_{22}^2, \tag{4. 6b}$$

where T_{11}^i, T_{12}^i, T_{21}^i and T_{22}^i (i = 1, 2) are defined in (3. 4), then there exists the coordinating strategy γ_c^* to coordinate two subsystems.

$$\gamma_c^* = -R_2^{i\,-1}[C_1^{i\,T}\ C_2^{i\,T}]K_1^{i*}(T^i)^{-1}\eta,\quad i=1 \text{ or } i=2, \tag{4. 7}$$

where K_1^{i*} is defined in (4. 3) and F^i is substituted by F^{i*}.

Similar to the finite-horizon problems, we require $2(n-r)(n-r) \geqq n\times 1$ to make the problems solvable.

5. Conclusion

In this paper, we have discussed the coordination problems for a class of two-level descriptor systems. The sufficient conditions for the existence of the descriptor variable feedback coordinating strategy are found for both the finite and infinite-horizon problems when N = 2. Extension to the cases when there are more than two subsystems is direct without any conceptual difficulty if there are a small number of dynamic modes in each subsystem. It should be emphasized that there exist no such a coordinating strategy for the similar problems formulated in the state-space systems. Since the descriptor variable model is suitable for describing large-scale systems consisting of a number of interconnected subsystems, it is expected to develop the method of this paper further to treat with more complex problems of large-scale systems.

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Appendix (Proof of Lemma 3. 1)

Substitute (2. 11) and (3. 1) into (2. 6) re-spectively, we get

$$\dot{x}_{i1} = A^i_{11}x_{i1} + A^i_{12}x_{i2} - W^i_1P^i(t)x_{i1} - W^i_2L^i_2(t)x_{i1} + W^i_2FL_{i1}(t)x_{i1} - W^i_2Fx_{i2}, \quad i=1,2, \tag{1a}$$

$$A^i_{21}x_{i1} + A^i_{22}x_{i2} - W^{iT}_2P^i(t)x_{i1} - W^i_3L^i_2(t)x_{i1} + W^i_3FL_{i1}(t)x_{i1} - W^i_3Fx_{i2} = 0, \quad i=1,2, \tag{1b}$$

and

$$\dot{x}_{i1} = A^i_{11}x_{i1} + A^i_{12}x_{i2} - W^i_1P^i(t)x_{i1} - W^i_2L^i_2(t)x_{i1} + W^i_{22}F^i(t)L^i_1(t)x_{i1} - W^i_{22}F^i(t)x_{i2}, \quad i=1,2, \tag{2a}$$

$$A^i_{21}x_{i1} + A^i_{22}x_{i2} - W^{iT}_2P^i(t)x_{i1} - W^i_3L^i_2(t)x_{i1} + W^i_{32}F^i(t)L^i_1(t)x_{i1} - W^i_{32}F^i(t)x_{i2} = 0, \quad i=1,2, \tag{2b}$$

where

$$W^i_{22} = C^i_1R^{i\,-1}_2C^{iT}_2, \quad W^i_{32} = C^i_2R^{i\,-1}_2C^{iT}_2, \qquad i=1,2. \tag{3}$$

Assumne that x^*_{i1} and x^*_{i2} are the unique team-optimal trajectories of S_i, then they must satisfy (1) according to Lemma 2. 1. From Lemma 2. 2, we can find that x^*_{i1} and x^*_{i2} are also the solutions of (2) no matter what $F^i(t)$ is. Since the Assumption 3. 1 is satisfied, there exists time-varing matrix $F^i(t)$ such that $A^i_{22} - W^i_{32}F^i(t)$ invertible for $t \varepsilon [t_0, t_f]$. Therefore, the solutions of (2) are uniquely determined, which must equal x^*_{i1} and x^*_{i2}. This fact means that (3. 1) constitute the team-optimal controls for two subsystems respectively.