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Two Tests for Jumps in High Frequency Financial Time Series: Simulation and Empirical Application

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Abstract

We often observe significant discontinuous variations, so-called jumps, in financial time series but empirically it is not easy to distinguish between a large variation and a discontinuous jump. The two tests have been proposed for detecting jumps in continuous diffusion process by using discrete data of high frequency financial time series. One is proposed by Barndorff-Nielsen and Shephard (2006) and the other was proposed by Lee and Mykland (2008). The former test is aimed to see if a time series is a jump diff usion process. In other word it can detect whether the process contains jumps or not globally. On the other hand, the latter test can detect the local jump arrival time and the size of realized jump. In this article we briefly introduce the two tests, show the empirical applications results, and examine the performance and applicability of the two tests. Furthermore we examine the performance of LM test by Monte Carlo experiment and real data analysis in particular.

1. Introduction

We often observe significant discontinuous variations, so-called jumps, in financial time series but empirically it is not easy to distinguish between a large variation and a jump in continuous diffusion process. The two tests have been proposed for detecting jumps in continuous diffusion process by using discrete data in high frequency financial time series. One is proposed by Barndorff-Nielsen and Shephard (2006), abbreviated by BNS test and the other were proposed by Lee and Mykland (2008), abbreviated by LM test hereafter. BNS test is aimed to see if a time series is a diffusion process with jump by using the bipower variation. In other word, it can detect whether the process contains jumps but it does not aim to detect neither jump arrival time nor realized jump sizes. So BNS test may be called as 'global test'. On the other hand LM test can detect the local jump arrival time and the size of realized jump. So it may be called as 'local test' although it may be used as global test by repeating the test. In this article we show empirical examples of the two tests and examine the performance and applicability of them. Furthermore we examine the performance of LM test by Monte Carlo experiment and real data analysis in particular.

This paper is organized as follows: Section 2 introduces the bipower variation and the BNS test and then shows empirical analysis given by Lu, Kawai, and Maekawa (2010). Section 3 introduces the LM test. Section 4 examines the performance of the LM test by Monte Carlo experiment. Section 5 presents empirieal results of LM test by the high frequency time series of Yen/US dollar exchange rate and Nikkei225. Finally we conclude in Section 6.

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2. Barndorff-Nielsen and Shephard Test

First we briefly review Barndorff-Nielsen and Shephard (2005), to get a clear idea of the test for the existence of jumps in a continuous diffusion process.

Let the log-price of a single asset be written as Y_t for continuous time $t \geq 0$ where Y is assumed to be a semimartingale. Further, Y^c and Y^d denote the continuous and the discontinuous component of Y respectively. The quadratic variation of Y is defined by

$$[Y]_t = P\lim_{n\to\infty} \sum_{j=0}^{n-1} \left(Y_{t,j+1} - Y_{t,j}\right)^2$$

where Y is observed at discrete time t_k; $t_0 = 0 < t_1 < \cdots < t_n = t$ such that $\sup_j \left\{t_{j+1} - t_j\right\} \to 0$ as $n \to \infty$. Then BNS showed that the quadratic variation can be decomposed into the continuous and discontinuous parts as

$$[Y]_t = \left[Y^c\right]_t + \left[Y^d\right]_t$$

where $\left[Y^d\right]_t = \Sigma_{0\leq u\leq t} \Delta Y_u^2$ and $\Delta Y_t = Y_t - Y_{t-}$. ΔY_t are the jumps. If there are no jumps in the process $\left[Y^d\right]_t = [Y]_t - \left[Y^c\right]_t = 0$.

BNS considered a case where Y is a member of the Brownian semimartingale plus jump (BSMP) class:

$$Y_t = \int_0^t a_s ds + \int_0^t \sigma_s dW_s + \sum_{j=1}^{N_t} c_j$$

where the drift a_s and the volatility σ is cadlag, W_t is a standard Brownian motion, N is a simple counting process and c_j are non-zero random variables.

In this case $\left[Y^c\right]_t = \int_0^t \sigma_s^2 ds$ and $\left[Y^d\right]_t = \Sigma_{j=1}^{N_t} c_j^2$, so

$$[Y]_t = \int_0^t \sigma_s^2 ds + \sum_{j=1}^{N_t} c_j^2$$

To calculate these quantities from the actual data we introduce $Y_{\delta[t/\delta]}$ as the discretized version of Y, where $\delta > 0$ is a time interval and $[t/\delta]$ is the largest integer that does not exceed t/δ. For simplicity we let Y_δ stand for $Y_{\delta[t/\delta]}$. Defining the difference of Y_δ at time j as $y_j = Y_{j\delta} - Y_{(j-1)\delta}$, $j = 1, 2, \cdots, [t/\delta]$. BNS introduced the realized quadratic and bipower variations which are respectively defined by

$$\left[Y_\delta\right]_t = \sum_{j=2}^{[t/\delta]} y_j^2,\ \left\{Y_\delta\right\}_t^{[1,1]} = \sum_{j=2}^{[t/\delta]} \left|y_{j-1}\right|\left|y_j\right|$$

Using these observable quantities it is shown that when $\delta \to 0$ the discontinuous part $\left[Y^d\right]_t = [Y]_t - \left[Y^c\right]_t$ can be consistently estimated from the actual data by

$$\sum_{j=2}^{[t/\delta]} y_j^2 - \mu_1^{-2} \sum_{j=2}^{[t/\delta]} \left|y_{j-1}\right|\left|y_j\right|$$

where $\mu_1 = E|u| = \sqrt{2}/\sqrt{\pi} \cong 0.79788$. Note that the continuous part $\left[Y^c\right]_t$ can be consistently estimated by $\mu_1^{-2} \Sigma_{j=2}^{[t/\delta]} \left|y_{j-1}\right|\left|y_j\right|$ without being affected jump components. This property is effectively utilized in the next section.

Using the above concepts and notations Barndorff-Nielsen and Shephard (2005) proposed three formulas of BNS tests,

$$\hat{G} = \frac{\delta^{-1/2}\left(\mu_1^{-2}\left\{Y_\delta\right\}_t^{[1,1]} - \left[Y_\delta\right]_t\right)}{\sqrt{\vartheta \mu_1^{-4}\left\{Y_\delta\right\}_t^{[1,1,1,1]}}} \xrightarrow{L} N(0,1)$$

(linear jump test)

$$\hat{H} = \frac{\delta^{-1/2}}{\sqrt{\vartheta\left\{Y_\delta\right\}_t^{[1,1,1,1]} / \left\{\left\{Y_\delta\right\}_t^{[1,1]}\right\}^2}}$$

$$\left(\frac{\mu_1^{-2}\{Y_\delta\}_t^{[1,1]}}{[Y_\delta]_t}-1\right)\xrightarrow{L}N(0,1)$$

(ratio jump test)

$$\hat{J}=\frac{\delta^{-1/2}}{\sqrt{\vartheta\max\left(t^{-1},\{Y_\delta\}_t^{[1,1,1,1]}\Big/\left\{\{Y_\delta\}_t^{[1,1]}\right\}^2\right)}}=\left(\frac{\mu_1^{-2}\{Y_\delta\}_t^{[1,1]}}{[Y_\delta]_t}-1\right)\xrightarrow{L}N(0,1)$$

(adjusted jump test)

where $\vartheta=(\pi^2/4)+\pi-5\cong 0.6090$ and $\{Y_\delta\}_t^{[1,1,1,1]}$ is defined by

$$\{Y_\delta\}_t^{[1,1,1,1]}=\delta^{-1}\sum_{j=4}^{\lfloor t/\delta\rfloor}|y_{j-3}||y_{j-2}||y_{j-1}||y_j|$$

Then it is shown

$$\{Y_\delta\}_t^{[1,1,1,1]}\xrightarrow{p}\mu_1^4\int_0^1\sigma_s^4 ds.$$

Therefore $\mu_1^{-4}\{Y_\delta\}_t^{[1,1,1,1]}\xrightarrow{p}\int_0^1\sigma_s^4 ds$ (consistent)

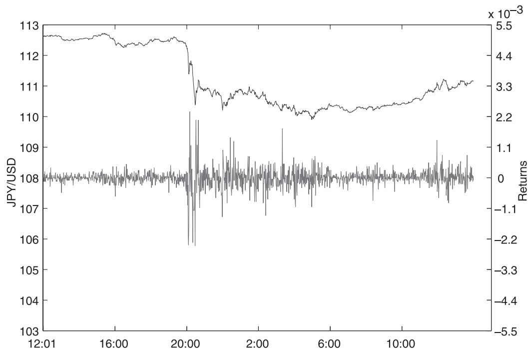
$\hat{J}$ test is devised so as to assure the positivity of denominator in H test by restricting

$$\{Y_\delta\}_t^{[1,1,1,1]}\Big/\left\{\{Y_\delta\}_t^{[1,1]}\right\}^2\geq 1/t.$$

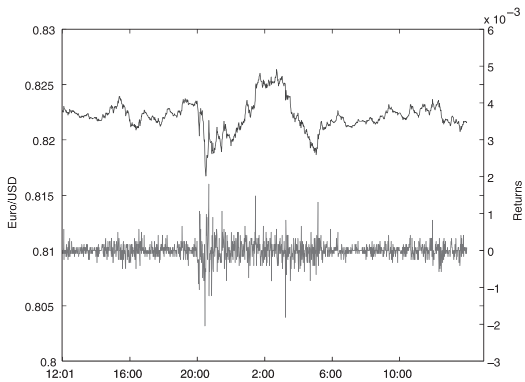
Monte Carlo simulation by BNS suggested that the adjusted ratio jump test J performed best.

Lu Xinhong, K. Kawai, and K. Maekawa (2009) applied BNS J-test for a high frequency time series of exchange rate of Japanese Yen, Euro, Australian dollar, Canadian dollar, Pound sterling against US dollar and detected jumps in these exchange rates when the Chinese Yuan was revaluated on 21, July 2005. See the following figures.

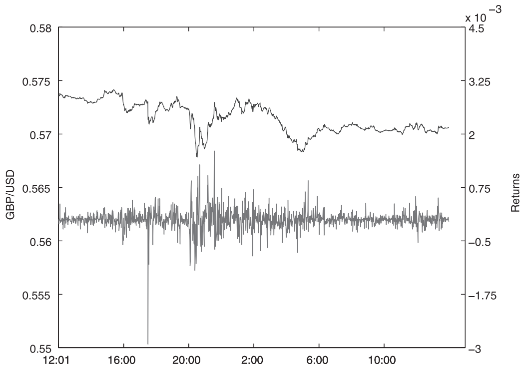
The results of J-test obtained by Lu et.al were cited in Table 1 below, from which we can



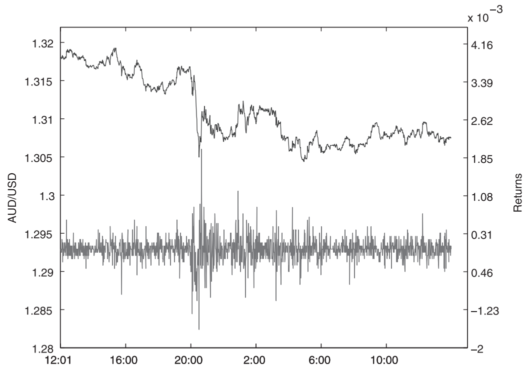
Japanese Yen/US Dollar and returns



Euro/US Dollar and returns



Pound Sterling/US Dollar and returns



Australian Dollar/US Dollar and returns

Table 1 Results of BPV J test for exchange rate

Japanese Yen	J = –3.16
Euro	J = –2.53
Australian Dollar	J = –5.38
Canadian Dollar	J = –4.24
Pound sterling	J = –4.30

5% critical value is –1.28 for J-test.

see that these five time series of exchange rates contained jumps. However as stated earlier BNS test is a global test to see if the process is a jump diffusion process overall, but it cannot detect a jump arrival time.

3. Lee and Mykland Test (LM Test)

In this section we briefly review Lee, S. and Mykland, P. A. (2008) test to get a clear idea and intuition of it. The BNS jump test aims to detect if the underlined process is a jump process overall. In this sense it may be call as a global test. On the other hand LM test statistic is a new nonparametric test to detect jump arrival time and realized jump sizes at a specific time. In this sense it may be called as a local test.

LM test considers the following model under suitably defined probability space. Let $W(t)$ be a standard Brownian motion, $\mu(t)$ be the drift and $\sigma(t)$ be the spot volatility. It is assumed that observation times $0 = t_0 < t_1 < \cdots < t_n = T$ are equally spaced: $\Delta t = t_i - t_{i-1}$ for simplicity, and that the drift and the volatility do not change dramatically over a short time period Δt. It should be noted that the frequency of observation is not necessarily so called high frequency, but is assumed relatively high frequency data compared to the length of observation period. Therefore 1 week, 1 day, 1 hour, 30 minutes, 15 minutes, 5 minutes can be regarded as high frequency if the length of observation period is long enough. A log return $\log S(t)$ is assumed to be generated by

$d\log S(t) = \mu(t)dt + \sigma(t)dW(t)$ when there are no jumps,

and generated by

$d\log S(t) = \mu(t)dt + \sigma(t)dW(t) + Y(t)dJ(t)$ when there are jumps,

where $J(t)$ is a counting process independent of $W(t)$ and $Y(t)$ is the jump size.

To get the intuition behind the LM test we cite the followings stated in LM (p.2539):

> *"Imagine that asset prices evolve continuously over time.*
> *Suppose that a jump occurs in a market at some time, say t_i. We would expect the realized asset return at that time to be much greater than usual continuous innovations. What if the spot volatility at that time is also high? Even if there is no jump, if the volatility is high and if we can only observe prices in discrete times, the realized return we observe may be as high as the return that actually due to jump. To distinguish those two cases, it is natural to standardize the return by a measure that explains the local variation only from the continuous part of the process. We refer to this measure as instantaneous volatility in this article and denoted it as $\sigma(t_i)$."*

To formulate this intuition LM test is constructed for discrete times and for a small interval of time $I_i = (t_{i-1}, t_i]$ by

$$T_i(\gamma) = \frac{\Delta_i S - \hat{\mu}_i(\gamma)}{\hat{\sigma}_i(\gamma)}$$

where

$$\Delta_i S = \log S_i - \log S_{i-1}$$

i.e., (logarithmic rate of return),

$$\hat{\mu}_i(\gamma) := \frac{1}{K-1}\sum_{J=i-K+1}^{i-1} \Delta_j S,$$

(estimator of local average of return),

$$\hat{\sigma}_i(\gamma) := \left(\frac{1}{K-2}\sum_{J=i-K+1}^{i-1} |\Delta_{j-1}S||\Delta_j S|\right)^{1/2},$$

(estimator of local volatility for constant γ, $0.5 < \gamma < 1$).

LM recommended choosing the window size by $K_n = O(n^\gamma)$.

Note that local volatility is estimated by $\hat{\sigma}_i(\gamma)$, which is called as realized bipower variation. It is based on realized data because the bipower variation has been shown to be consistent estimator for the integrated volatility, even when there are jumps in return process. The validity of using the realized bipower variation $\hat{\sigma}_i(\gamma)$ is stated as *"Despite the intuition that jumps in a process may impact its volatility estimation, it remains consistent no matter how large or small jumps are mixed with the diffusive part of pricing models"* (see p. 2539 in LM paper).

Thus the LM test statistic has been introduced. Next in order to apply this test we have to know the critical values of the distribution of the test statistic. Concerning the asymptotic distribution of it the following properties have been shown. LM showed that $T_i(\gamma)$ is approximately decomposed as jump part and without jump part, which has been proved .

$$T_i(\gamma) \cong T_i^0(\gamma)$$

when there are no jumps and

$$T_i(\gamma) \cong T_i^0(\gamma) + \frac{\Delta_i Z}{c_2 \sigma_{t_{i-K}} \sqrt{h}}$$

when there are jumps, where,

$$T_i^0(\gamma) := \frac{1}{c_2}\left(U_i - \frac{1}{K-1}\sum_{j=i-K+1}^{i-1} U_j\right)$$

and $h = t_i - t_{i-1}$, $c_2 = \sqrt{2/\pi}$, $U_i \sim \mathrm{iidN}(0,1)$.

The distribution of $T_i(\gamma)$ is approximately by $T_i^0(\gamma,2)$ when there are no jumps and $\Delta t \to 0$. The asymptotic distribution of the $\max_{i \le n}|T_i^0(\gamma,2)|$ is given by

$$\lim_{n\to\infty} \Pr\left[b_n\left(\max_{i\le n}|T_i^0(\gamma)| - a_n\right) \ge x\right] = 1 - \exp(-e^{-x})$$

where,

$$a_n = \frac{\sqrt{2\log n}}{c_2} - \frac{\log\pi + \log\log n}{2c_2\sqrt{2\log n}},$$

$$b_n = c_2\sqrt{2\log n},\ c_2 = \sqrt{\frac{2}{\pi}}$$

$1 - \exp(-e^{-x})$ is a significance level α such as $\alpha = 0.05$. From this the threshold value is given by $\beta = -\log\{-\log(1-\alpha)\}$ and

If $b_n\left(|T_i(\gamma)| - a_n\right) > \beta$, there is a jump in the time period $I_i = \left(t_{i-1}, t_i\right]$.

If $b_n\left(|T_i(\gamma)| - a_n\right) \le \beta$, there is no jump.

Summary of the calculation steps of LM test

(1) Determine the window size: $K_n = O(n^\gamma)$. If the frequency of observation is 1 week, 1 day, 1 hour, 30 minutes, 15 minutes, 5 minutes, the window size is recommended to take 7, 15, 78, 110, 156, 270 separately.

(2) Calculate the local average of return $\hat{\mu}_i(\gamma)$, the local volatility $\hat{\sigma}_i(\gamma)$, and then calculate LM test statistic $T_i(\gamma)$.

(3) Choose the significance level α, and then calculate the threshold value

$$\beta = -\log\{-\log(1-\alpha)\}$$

(4) If $b_n\left(|T_i(\gamma)| - a_n\right) > \beta$, then we may conclude that there is a jump in the time period $I_i = \left(t_{i-1}, t_i\right]$. If $b_n\left(|T_i(\gamma)| - a_n\right) \le \beta$, then we may conclude that there is no jump.

4. Empirical Applications

This section shows two empirical examples of LM test as follows.

4.1 Example 1: Japanese Yen/US Dollar Exchange Rate High Frequency Data

We apply LM test for the 5 minutes high frequency data in Yen/Dollar exchange rate in the period of the worldwide financial crisis from 2008/06/01 to 2008/11/30. Figure1 shows the transition of logarithmic return of high frequency (Δt = 5 minutes) data of the Yen/US Dollar exchange rate in the above mentioned period with 38,332 observations. This figure has a typical feature of a GARCH process with large jumps. We apply the LM test to this data to detect if there are jumps. We used four significance levels such as α = 0.05, 0.01, 0.001, 0.0001. The results are shown in Table 2 and Figure 1.

Table 2 Jump Frequency in 5 Minutes Interval

α	N	N (jumps)	JF (jumps)
0.05	38,332	64	0.0017
0.01	38,332	52	0.0014
0.001	38,332	41	0.0011
0.0001	38,332	27	0.000704

In Table 2 N is the number of the observations, N (jumps) the number of jumps, and JF (jumps) = N (jumps)/N. Table 2 clearly shows that jumps were detected for all significance levels. Figure 1 shows economic event corresponding extremely large jumps denoted by ①, ②,③. They correspond to the following events: ① On September 14, 2008, Bank of America

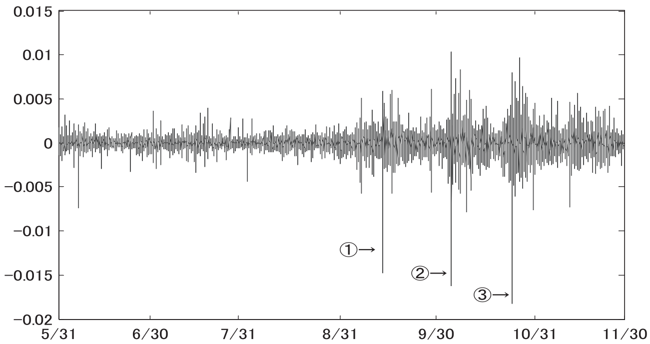


Figure 1 Logarithmic Return of 5-min Yen/US Dollar Exchange rate from 2008/06/01 to 11/30

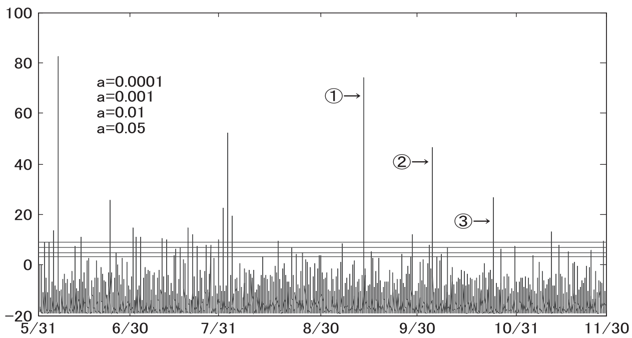


Figure 2 Transition of LM-test Statistics Corresponding to the Figure 1 Series

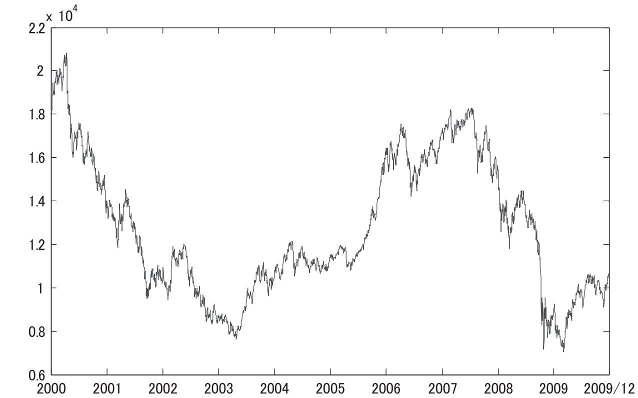


Figure 3 Nikkei225 Series, 2000/01/01-2009/12/30

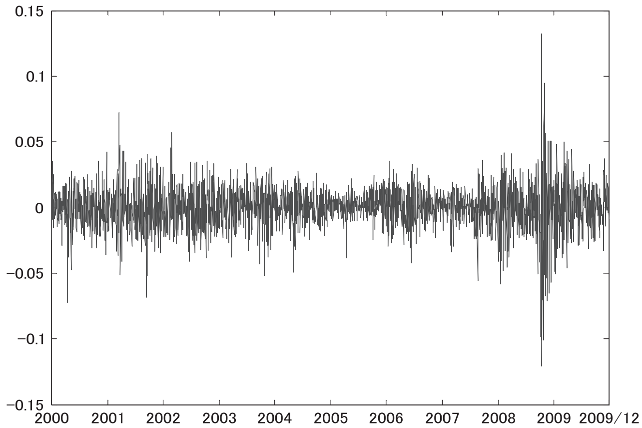


Figure 4 Logarithmic Return Series

announced an agreement to acquire Merrill Lynch.

② On October 6, 2008, Iceland's prime minister declared financial emergency.

③ On October 24, 2008, IMF announced emergency loan to Iceland.

Figure 2 shows transition of LM-test statistics. The four horizontal lines from the bottom to the top in the figure correspond to the four significant levels: $\alpha = 0.05, 0.01, 0.001, 0.0001$.

These results seem that the LM test can appropriately detect jump arrival times associated with some economic events.

4.2 Example 2: Daily Return of Nikkei225

As mentioned in the previous section, daily data can be treated as high frequency data if the length of observation period is long enough. In this section we deal with daily data of Nikkei 225 during the period from 2000/01/01 to 2009/12/30 with 2,545 observations, shown in Figure 3, which are long enough to apply LM test. Figure 4 is the transition of the log return of it, which seems to have volatility clustering of GARCH type with occasional jumps. We applied LM test to the log-return and the results are shown in Table 3 and Figure 5.

Figure 5 shows the transition of the values of LM test statistic in which the four horizontal

Table 3 Jump frequency of Nikkei225 Daily data In the period of 2000/01/01-2009/12/30

$lm\alpha$	N	N (jumps)	JF (jumps)
0.05	2,454	5	0.002
0.01	2,454	4	0.0016
0.001	2,454	2	8.15E-04
0.0001	2,454	0	0

N is the number of the observations,
N (jumps) the number of jumps,
JF (jumps) = N (jumps)/N

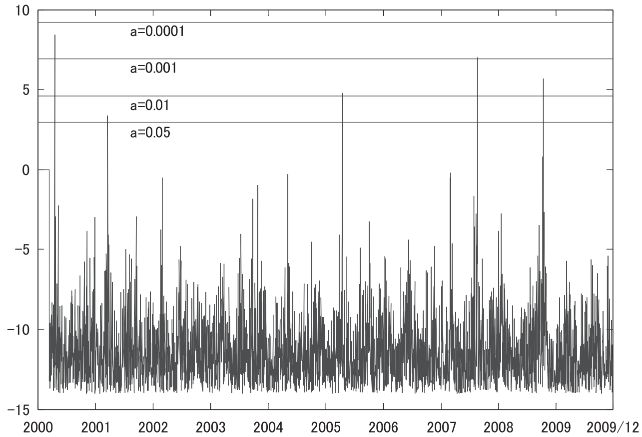


Figure 5 LM Test Result

lines from the bottom to the top correspond to the four significant levels: $\alpha = 0.05$, 0.01, 0.001, 0.0001.

On the contrary to the exchange rate series in the previous case, large jumps in this case are not easy to be associated with some economic events.

5. Simulation Study

This section shows the results of a Monte Carlo experiment. We use the same the data generating process (DGP) as in Masuda and

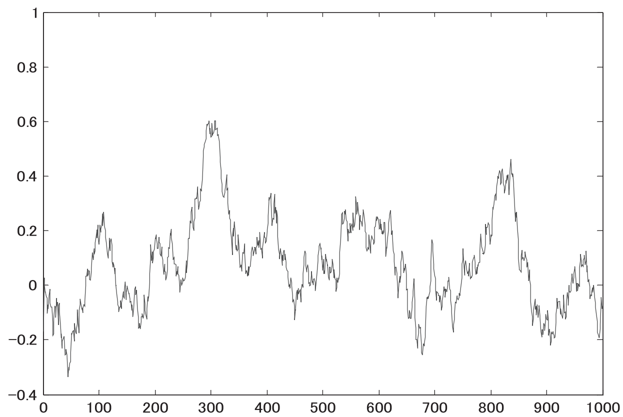


Figure 6 A Generated Series Obtained by $x(t) = -0.2tdt + w(t)$ When the Length of Series is $n = 1{,}000$ and $\delta^2 = 0.01$.

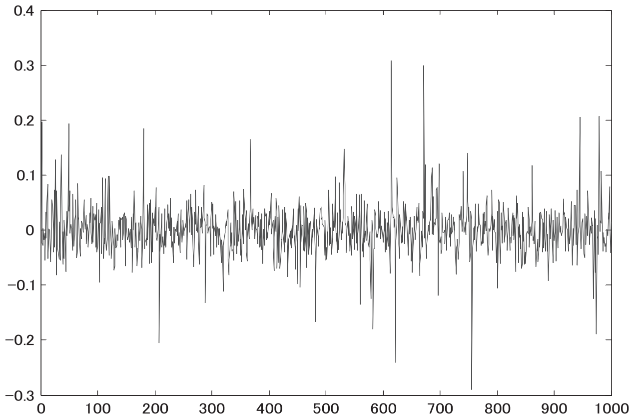


Figure 7 A Generated Return Series for n = 1,000 with Jumps

Table 4 Simulation Results of LM Test when $\alpha = 0.05$, $\delta^2 = 0.01$.

Number of iteration	n	k	mean (J)	s.d. (J)	mean (mRV)	s.d. (mRV)
1,000	5,000	70	0.3847	0.046	1.1166	0.032
1,000	10,000	100	0.5346	0.0478	1.0428	0.0177
1,000	20,000	141	0.6531	0.0486	1.0145	0.0115

n: number of observations, k: window size,
Mean (J): mean ratio of success out of 1,000 iterations,
s.d. (J): standard deviation of the number of successes
mean (mRV): mean of the modified RV, denoted by mRVn[1)]
s.d. (mRV): standard deviation of the modified RV, denoted by mRVn
[1)] mRV is defined by

$$mRV_n := \frac{n}{n - \#\hat{J}_n} \sum_{j<n, j \notin \hat{J}_n} \left(\Delta_j X\right)^2 \quad \text{(See Masuda and Morimoto (2009))}$$

where
$\hat{J}_n$: a set of i corresponding to an interval $I_i(\frac{i-1}{n}, \frac{i}{n}]$ in which a jump occurred.
$\#\hat{J}_n$: a number of the intervals in which a jump occurred.
Hence mRV_n is the sum of square of returns after deleting the intervals in which jumps occurred, in other word, it is the continuous part of the cumulated volatility.

Morimoto (2009):

$$x(t) = -0.2t + w(t) + z_t$$

where z_t is a jump and $w(t)$ is a standard Brownian motion. z_t and $w(t)$ are assumed to be independent. The number of jumps is fixed in 100 throughout our experiment. The jump sizes are assumed to be normally distributed $N(0,\delta^2)$. Figure 6 is an example of a series picked up from 1,000 generated series (number of iterations denoted by loop hereafter) each of which has n = 1,000 observations. In our simulation we generated 1,000 paths, for n = 5,000, 10,000, 20,000. These simulation results are shown in Table 4 and Figure 7. From the results we can see that LM test can detect jumps fairy well. But the ratio of detecting jumps is rather low compared with Masuda and Morimoto (2009). This might be due to our generating scheme of jumps which is different from preceding papers.

6. Concluding Remarks

Jumps are often observed in high frequency financial time series. If there are jumps in a return process in a stock market and if we can identify the process as a jump diffusion process, then risk management may be changed accordingly. Therefore detecting and testing for jumps are very important. Recently new tests were proposed. One is the test proposed by Barndorff-Nielsen and Shephard (2006) which is aimed to test if a process contains jumps somewhere in the process. We call this test as a global test (abbreviated BNS test). The other test is proposed by Lee, S. and Mykland, P. A. (2008) which is aimed to detect jump arrival times and jump sizes. By this nature we call this test as a local test (abbreviated by LM test). In this paper we have examined those two tests by empirical analysis and Monte Carlo experiment. From the results of our analysis we have reconfirmed that the BNS and LM tests show good performance.

Finally we conclude this paper by making a following comment on an application of LM test. Lu et.al (2009) applied the BNS test to detect jumps in the five return series of foreign exchange rates series. But since the BNS test could not identify exact jump arrival times Lu et.al (2009) regarded a large change in return as a jump when the magnitude of changes exceed an arbitrarily given size. They collected such jumps in the five returns series and showed that jumps in the deferent currencies are correlated. But since their criterion seems more or less subjective it may be desirable to detect jumps by a more objective criterion for jumps. Instead the LM test could be used as an objective criterion to detect the jump arrival time and it could give us more reliable correlation analysis of jumps in different return series. We will try such correlation analysis based on LM test in the future.

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