Mathematical Solution and Numerical Simulation of Human Bioenergetics

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INTRODUCTION

Three component hydraulic model proposed by Margaria (1976) described the human energy process during exercise. Oxygen consumption, lactate formation and phosphagen breakdown are modeled after the three interconnected vessels in the system. Morton (1986) solved this model mathematically on the basis of experimental data. It is what we call M-M (Margaria-Morton) model, and is the generlization of Margaria's original.

The purpose of this study is to analyze the anaerobic power output during a strenuous exercise by the use of M-M model.

METHOD

24 years male subject participated in our study. His weight and LBM were 55.0 kg and 48.7 kg. The maximal effort cycling bout were performed on a Monark bicycle ergometer with a resistive loading from 2.0 to 3.0 kgω. A microswitch with dry battery was fixed on near the wheel of the ergometer. A pulse was generated with every rotation of wheel, and recorded by the data recorder (TEAC; R-40). By using the A/D converter (CONTEC; AD-12-16S-(98)), numerical data was

calculated on the 16 bits computer (NEC; PC-9801).

M-M MODEL

Figure 1 below gives a diagrammatical representation of the M-M model.

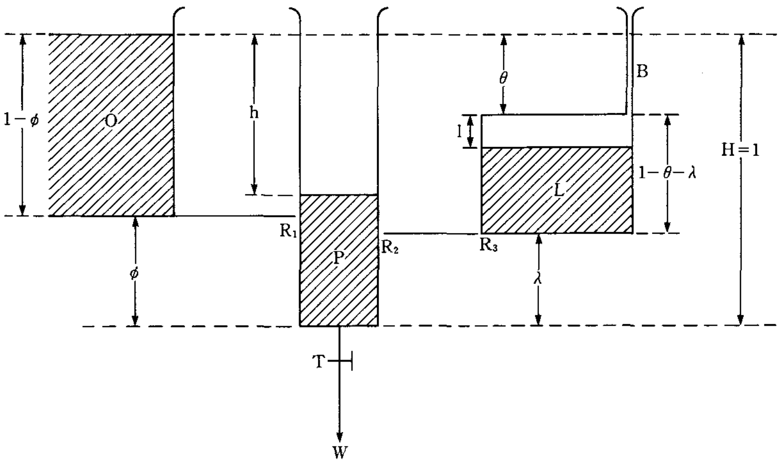

Fig. 1 Margaria-Morton model

Three vessels O, P, and L representing the oxidative source, the phosphagen, and the glycolytic source. The height of the base of vessel O above the base of vessel P is denoted by ϕ. The top of vessel L is at a level θ below the fluid level in vessel O. 'h' is a decreased fluid level of vessel P. And Wm represents the maximal energy expenditure.

MATHEMATICAL SOLUTION

In the first phase of activity, the fluid in P has dropped to the level $h<\theta$. The form of equation is given by:

$$Wm = Mp\ (1-h) = Mo\ \{h/\ (1-\theta)\} + Ap\ (dh/dt)$$

where: $Mp = \dot{V}pmax$, the maximal flow from vessel P

$Mo = \dot{V}o2\ max$, the maximal flow from vessel O

Ap = cross sectional area of vessel P

This is a simple first order linear differential equation having as its general solution:

$$h = Mp/\{Mo/(1-\phi)+Mp\}\ [1+C\cdot exp\{-(1/Ap)\ (Mo/(1-\phi)+Mp)t\}]$$

The paticular solution (h=0 at t=0) is given by:

$$h = Mp/\{Mo/(1-\phi)+Mp\}\ [1-exp\ \{-1/Ap\ (Mo/(1-\phi)+Mp)t\}]$$

$$Wm = Mp\ [\{1-Mp/(Mo/(1-\phi)+Mp)\} + \{Mp/(Mo/(1-\phi)+Mp)\} * exp\ \{-1/Ap\ (Mo/(1-\phi)+Mp)\}t]$$

$$\dot{V}o2 = Mo/(1-\phi)\ [Mp/\{Mo/(1-\phi)+Mp\}] * [1-exp\ \{-1/Ap\ (Mo/(1-\phi)+Mp)t]$$

$$\dot{V}p = Wm - \dot{V}o2$$

when $h=\theta$ (The end of first phase):

$$t1 = Ap/\{Mo/(1-\phi)+Mp\}\cdot ln[1-(1/Mp)\ \{Mo/(1-\phi)+Mp\}\theta]$$

The general form is given by:

$$Wm = \alpha\{\beta+(1-\beta)\cdot exp\ (-\delta t)\}$$

By using Runge-Kutta-Gill method, the numerical simulation was done after second phases.

RESULTS and DISCUSSIONS

Figure 2 showed, there was much difference between the simulation and the experimental data. Figure 3(a) showed the simulated total of Wm, $\dot{V}o\ 2$, $\dot{V}p$ and $\dot{V}\ 1$. Figure 3(b) Showed % of totals as such.

Second attempt, the shape of P vessel was transformed into a funnel shape, there was then little difference between them (fig. 4). Absolute

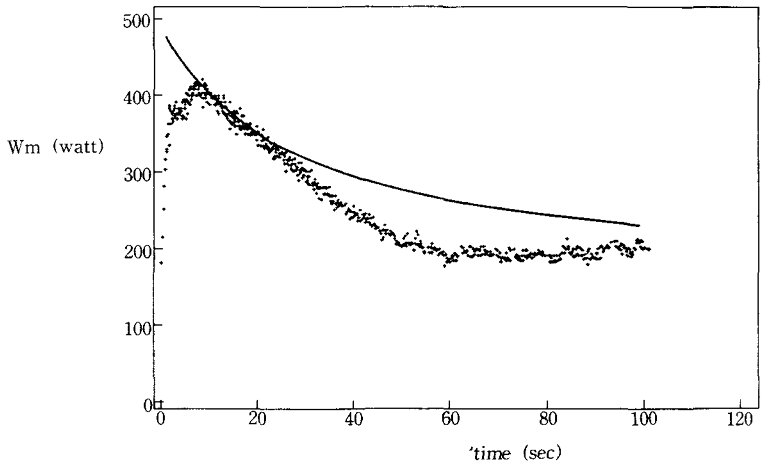

Fig. 2 Wm of simulation and experimental data (first attempt)

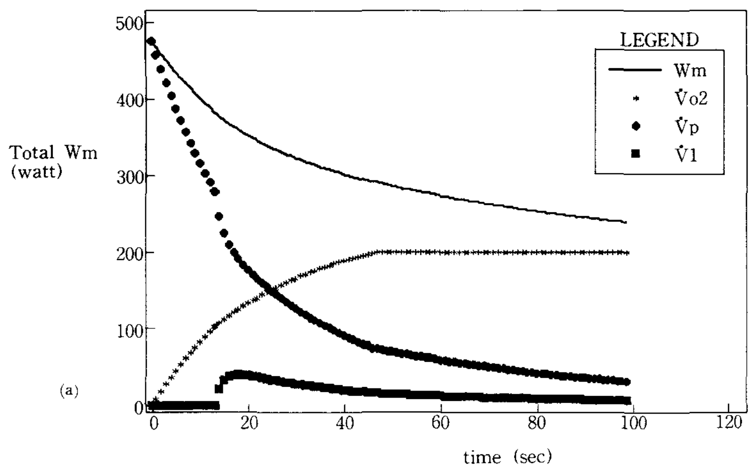

Fig. 3(a) Simulated total of Wm, Vo 2 Vp, and Vl (a). and % of totals as such (b). (first attempt)

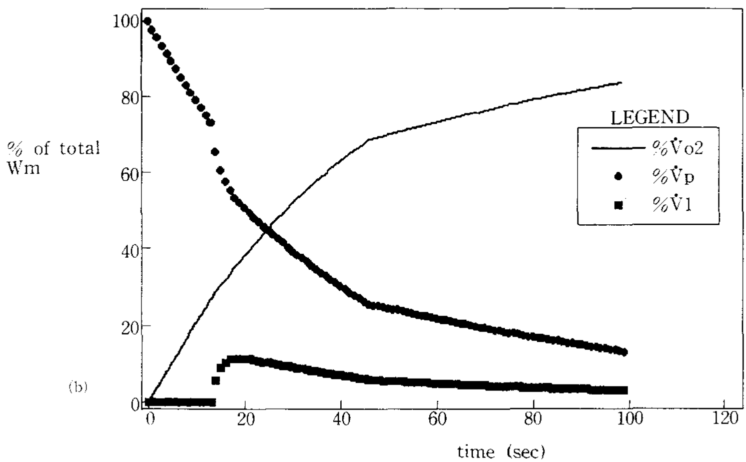

Fig. 3(b) Simulated total of Wm, Vo 2 Vp, and Vl (a). and % of totals as such (b). (first attempt)

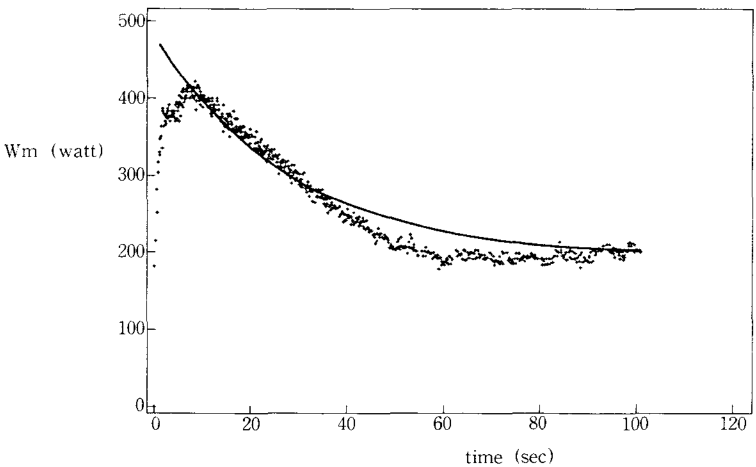

Fig. 4 Wm of simulation and experimental data (second attempt)

totals and % of totals were shown in the figures 5(a) and 5(b).

Third attempt, the position of O vessel and L vessel was replaced each other, it was reconnized that there existed no more difference between them (fig. 6 fig 7(a) and 7(b).

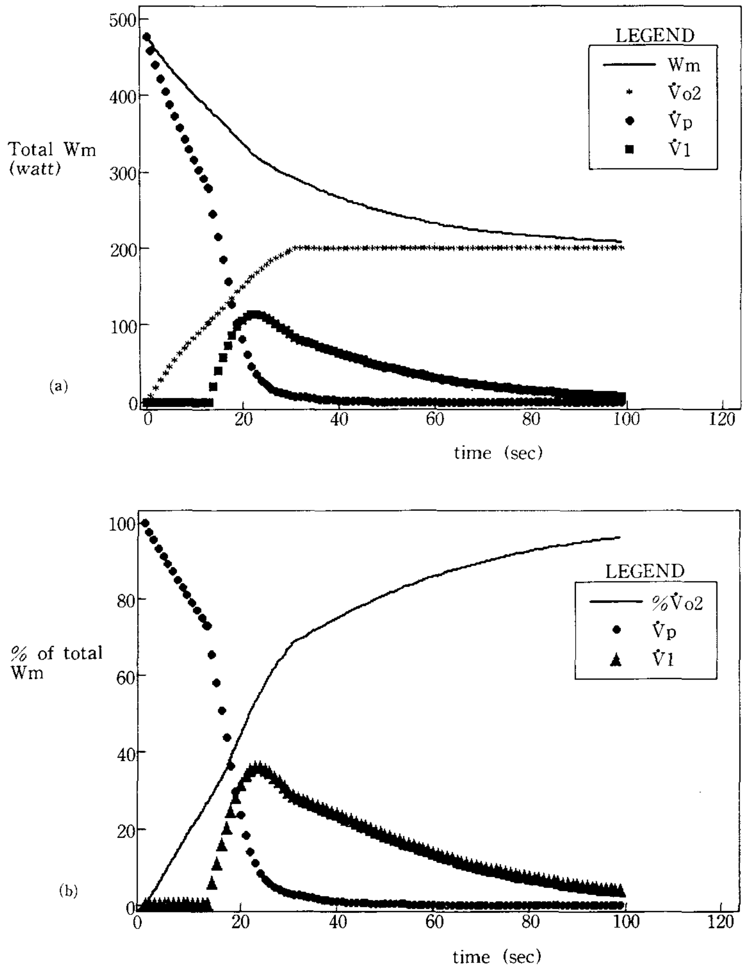

Fig. 5 Simulated total of Wm, Vo 2, Vp and Vl (a). And % of totals as such (b).

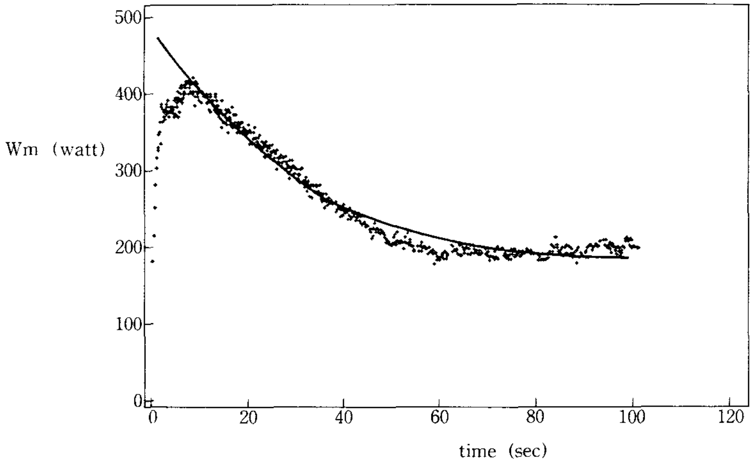

Fig. 6 Wm of simulation and experimental data (third attempt)

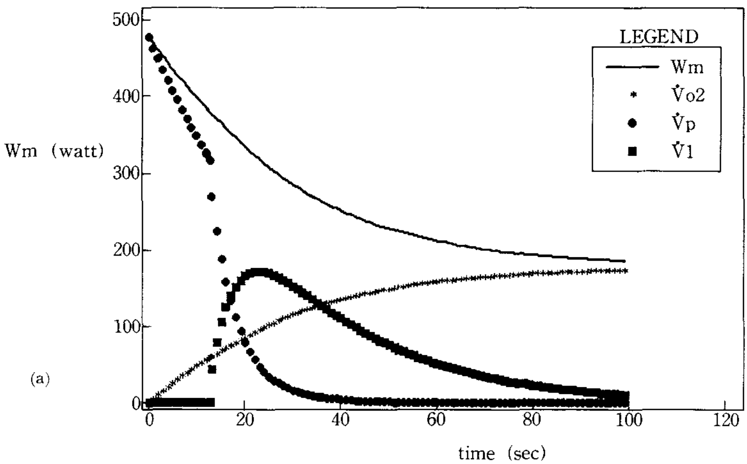

Fig. 7(a) Soimulated total of Wm, Vo 2, Vp and Vl (a). And % of totals as such (b).

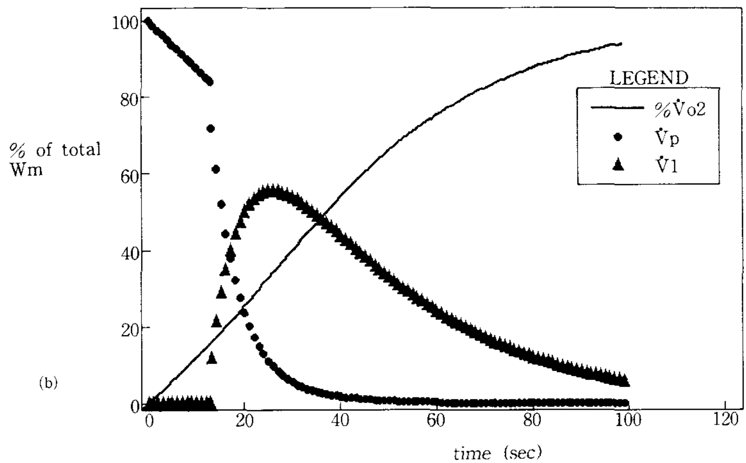

Fig. 7(b)　Soimulated total of Wm, Vo 2, Vp and Vl (a).　And % of totals as such (b).

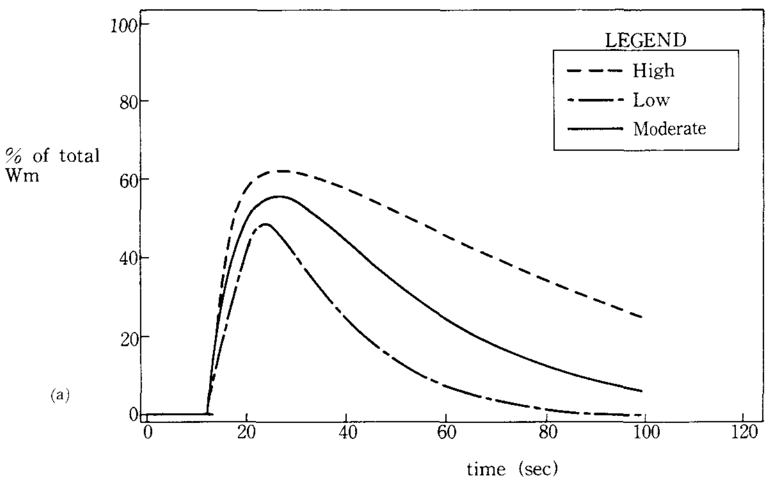

Fig. 8(a)　Changed output level of L vessel (a).　And Simulation of glycolytic power effect (b)

(b)

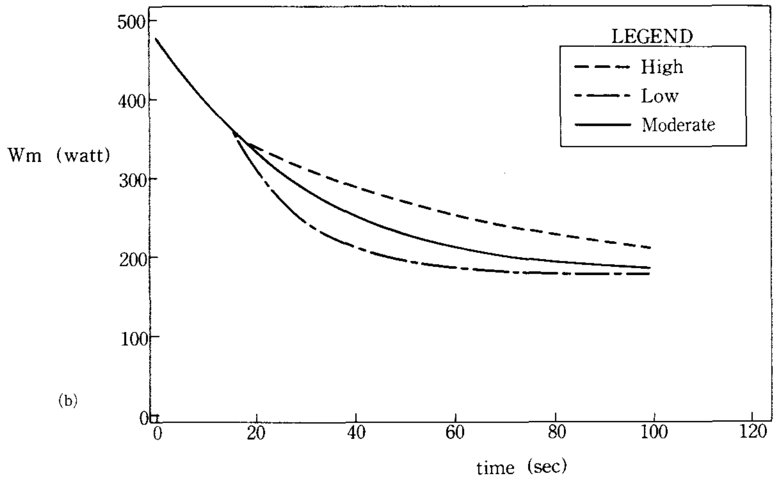

Fig. 8(b) Changed output level of L vessel (a). And Simulation of glycolytic power effect (b)

Furhter more, an output level of L vessel could also be changed artificially (fig. 8(a)). Figure 8(b) showed there was simulation of a glycolytic power effect. It may be safely said, therefore, that Wingate test and the other anaerobic power test estimate the glycolytic energy flow accurately.

References

1) Margaria R (1976) Biomechanics and energetics of muscular exercise. Oxford University Press, Oxford.

2) Morton R. H. (1986) On a model of human bioenergetics 2. Eur. J. Appl. Physiol., 55, 413-418.

3) Morton R. H. (1986) A three component model of human bioenergetics. J. Math. Biol. 24, 451-466.

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